13.3 Arithmetic and Geometric Series and Their Sums

Series - The sum of the terms of a sequence.

Sequence: 1, 5, 9, 13
Series: 1 + 5 + 9 + 13

Sum of a finite Arithmetic Sequence

S_n - sum of the first n terms in a series.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

1) Find the sum of the first 30 terms of $5 + 9 + 13 + 17 + \ldots$

$n = 30$
$a_1 = 5$
$a_{30} = 121$

$a_{30} = 5 + 4(30 - 1)$
$a_{30} = 121$

$$S_{30} = \frac{30}{2}(5 + 121)$$

$$S_{30} = 1890$$

Sum of a finite Geometric Series

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

1) Find the sum of the first ten terms of the geometric series.

$4+8+16+32+\ldots$

$a_1 = 4$
$r = 2$
$n = 10$

$$S_{10} = 4\left(\frac{1-(2)^{10}}{1-2}\right)$$

$$S_{10} = 4092$$

Sum of an infinite Geometric Series

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right) \qquad r<1$$

$$S_n = a_1\left(\frac{1-\left(\frac{1}{4}\right)^{n\to 0}}{1-\left(\frac{1}{4}\right)}\right) \qquad n \text{ is really big}$$

S_n ↑ heading to infinity

$$S_n = a_1\left(\frac{1}{1-r}\right)$$

$$\boxed{S = \frac{a_1}{1-r} \quad |r|<1}$$

Find the infinite sum of the series

~~$10, 5, \frac{5}{2}, \frac{5}{4},$~~

$10 + 5 + \frac{5}{2} + \frac{5}{4} + \ldots$

$$S = \frac{10}{1-\left(\frac{1}{2}\right)}$$

$$S = 20$$

Sigma Notation

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$$

n ← Stops

Sigma →

Start → k=1

Read "The sum of a_k from k=1 to n"
k is called the index.

1) Expand and find the sum.

$$\sum_{k=1}^{4} (2k+1)$$

$2(1)+1 = 3$
$2(2)+1 = 5$
$2(3)+1 = 7$
$2(4)+1 = 9$

Expanded: $3+5+7+9$
Sum: 24

Write the series using Sigma notation.

$5 + 11 + 17 + 23 + 29 + \ldots + 83$

(+6 between 5 and 11; a_n = 83)

$$\sum_{n=1}^{n} (\text{formula})^{a_n}$$

$a_n = 5 + 6(n-1)$

$a_n = 6n - 1$

$83 = 6n - 1$

$n = 14$

$$\sum_{n=1}^{14} (6n-1)$$

Find the sum of the first 100 terms. $\sum_{n=1}^{100} (6n-1)$

$$S_{100} = \frac{n}{2}(a_1 + a_n)$$

$$= \frac{100}{2}(5 + 599)$$

$a_1 = 6(1) - 1$ $\quad$ $a_{100} = 6(100) - 1$

$a_1 = 5$ $\quad$ $a_{100} = 599$

$$S_{100} = 30{,}200$$