

Sequences and Series

Sequence - A set of numbers written in a specific order.

Ex: 2, 4, 8, 16, 32, ...
 a_1 a_2 a_3 a_4 a_5

Notation - $a_1, a_2, a_3, \dots, a_{n-1}, a_n$

a_1 - first term

a_2 - 2nd term

a_3 - 3rd term

\vdots

a_n - nth term

Recursive Form - given the 1st term
- given the nth term (formula)

Ex: Find the first three terms of the given recursively defined sequence.

$$a_1 = 3$$

$$a_n = 2a_{n-1}$$

previous term

$$a_1 = 3$$

$$a_2 = 2a_{2-1}$$

$$a_2 = 2a_1$$

$$a_2 = 2(3)$$

$$a_2 = 6$$

$$a_3 = 2a_{3-1}$$

$$a_3 = 2a_2$$

$$a_3 = 2(6)$$

$$a_3 = 12$$

Ex: Find the first 4 terms of the given sequence.

$$a_1 = 4 \quad a_n = 3a_{n-1} + 2$$

$$\begin{array}{llll} a_1 = 4 & a_2 = 3a_1 + 2 & a_3 = 3a_2 + 2 & a_4 = 3a_3 + 2 \\ a_2 = 3(4) + 2 & a_3 = 3(14) + 2 & a_4 = 3(44) + 2 & \\ a_2 = 14 & a_3 = 44 & a_4 = 134 & \end{array}$$

Ex: Find the first 3 terms and the 1000th term

$$a_1 = 5 \quad a_n = 2a_{n-1} + 4$$

$$\begin{array}{llll} a_1 = 5 & a_2 = 2a_1 + 4 & ~~a_3 = 2(14) + 4~~ & a_3 = 2a_2 + 4 \\ a_2 = 2(5) + 4 & & & a_3 = 2(14) + 4 \\ a_2 = 14 & & & a_3 = 32 \end{array}$$

$$a_{1000} = 2a_{999} + 4$$



* Recursive Form - Dependent on previous terms
so it doesn't always make
sense to use it.

Explicit Form - A formula where n is substituted in to find the term.

Ex: Find the first 3 terms and the 10th term of each sequence.

1) $a_n = 2n + 1$

$a_1 = 2n + 1$	$a_2 = 2n + 1$	$a_3 = 2n + 1$	$a_{10} = 2n + 1$
$= 2(1) + 1$	$a_2 = 2(2) + 1$	$a_3 = 2(3) + 1$	$a_{10} = 2(10) + 1$
$a_1 = 3$	$a_2 = 5$	$a_3 = 7$	$a_{10} = 21$

2) $a_n = (-1)^n n^2$

$a_1 = (-1)^1 (1)^2$	$a_2 = (-1)^2 (2)^2$	$a_3 = (-1)^3 (3)^2$	$a_{10} = (-1)^{10} (10)^2$
$a_1 = -1$	$a_2 = 4$	$a_3 = -9$	$a_{10} = 100$