Module 5 Review Sheet: Piecewise Functions Name Key

#1-5 Identify the independent and dependent variables.

1) The cost, in dollars, to fill up my gas tank, in gallons, with unleaded fuel

Independent: # of gallons Dependent: Cost in $

2) The height of a humans as a function of age

Independent: Age Dependent: height

3) The price of a ribeye steak at Harris Teeter

Independent: # of pards Dependent: Cost

4) The amount of money in a paycheck of an hourly employee at Harris Teeter

Independent: # of hours Dependent: $

5) The price of a cheese pizza from Domino's

Independent: size Dependent: Cost

6) Evaluate

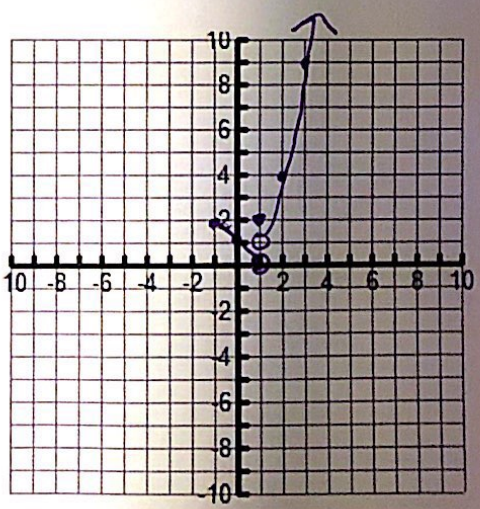
$$f(x)=\begin{cases} x^2-5 & -9<x\le -4 \\ \frac{1}{2}x+7 & -4<x<8 \\ -3 & x\ge 8 \end{cases}$$

a) $f(5)=$ 9.5 b) $f(-7)=$ 44 c) $f(12)=$ -3

d) $f(-15)=$ undefined e) $f(8)=$ -3 f) $f(4)=$ 9

#7 – 10 Sketch the graph of the following functions and answer the related questions.

7) $$f(x)=\begin{cases} -x+1 & -1\le x<1 \\ 2 & x=1 \\ x^2 & x>1 \end{cases}$$



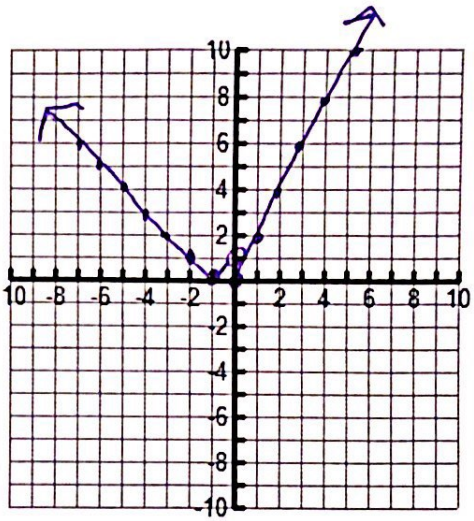
Domain: $[-1,+\infty)$

Range: $\in (0,+\infty)$

Interval(s) of increasing: $(1,+\infty)$

Interval(s) of decreasing: $\in (-1,1)$

8) $f(x) = \begin{cases} |x+1| & x < 0 \\ 2x & x \geq 0 \end{cases}$



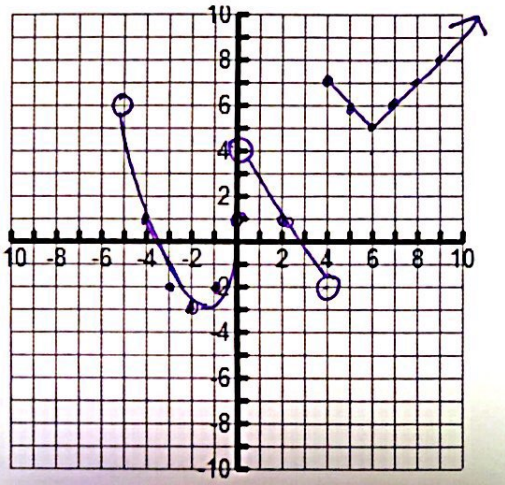
Domain: $(-\infty, +\infty)$

Range: $[0, +\infty)$

Interval(s) of increasing: $(-1,0) \cup (0, +\infty)$

Interval(s) of decreasing: $(-\infty, -1)$

9) $f(x) = \begin{cases} (x+2)^2 - 3 & -5 < x \leq 0 \\ -\frac{3}{2}x + 4 & 0 < x < 4 \\ |x-6| + 5 & x \geq 4 \end{cases}$



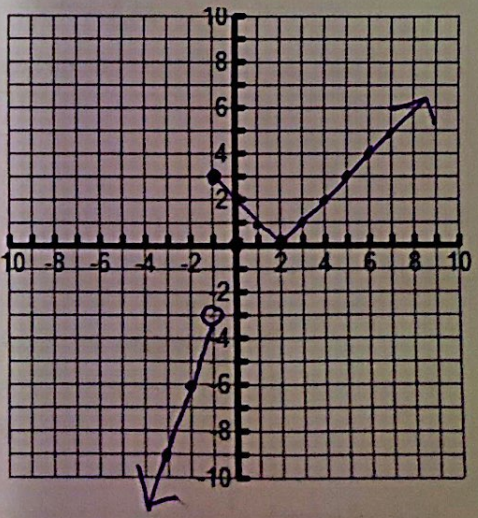
Domain: $[-5, +\infty)$

Range: $[-3, +\infty)$

Interval(s) of increasing: $(-1,0) \cup (6, +\infty)$

Interval(s) of decreasing: $(-5,-1) \cup (0,4) \cup (4,6)$

10) $f(x) = \begin{cases} 3x & x < -1 \\ |x-2| & x \geq -1 \end{cases}$



Domain: $(-\infty, +\infty)$

Range: $(-\infty, -3) \cup [0, +\infty)$

Interval(s) of increasing: $(-\infty, -1) \cup (2, +\infty)$

Interval(s) of decreasing: $(-1, 2)$

Solve each system of equations. Round to the nearest hundredth. Show work on your own paper.

11) $2x^2 + y^2 = 18$
$xy = 4$

12) $x^2 + y^2 = 4$
$y - x^2 = -9$

13) $x^2 - y^2 = 21$
$x + y = 7$

14) $x^2 + y^2 = 29$
$xy = 10$

Write a system of equations for each of the following situations. Then solve the system algebraically or graphically. Do not just guess check!

15) The sum of two numbers is 8 and the sum of their squares is 34. Find the numbers.

System of Equations: $x+y=8$ $x^2+y^2=34$ Solution: 3 & 5

$x=5$ $x=3$ $y=8-x$

$y=8-5$ $y=8-3$

$y=3$ $y=5$

$x^2+(8-x)^2=34$

$x^2+64-16x+x^2=34$

$2x^2-16x+64=34$

$2x^2-16x+30=0$

$2(x^2-8x+15)=0$

$2(x-5)(x-3)=0$

16) The sum of two numbers is 10 and the difference of their squares is 50. Find the numbers.

System of Equations: $x+y=10$ $x^2-y^2=50$ Solution: 7.5 & 2.5

$y=10-x$

$x^2-(10-x)^2=50$

$x^2-(100-20x+x^2)=50$

$x^2-100+20x-x^2=50$

$-100+20x=50$

$20x=150$

$x=7.5$

$y=10-7.5$

$y=2.5$

17) The product of two numbers is 24 and the sum of their squares is 73. Find the numbers.

System of Equations: $xy=24$ $x^2+y^2=73$ Solution: 3 & 8, -3 & -8

$xy=24$

$y=\frac{24}{x}$

$x^2+\left(\frac{24}{x}\right)^2=73$

$x^2+\frac{576}{x^2}=73$

$x^4+576=73x^2$

$x^4-73x^2+576=0$

$(x^2-9)(x^2-64)$

$x^2-9=0$ $x^2-64=0$

$x=\pm 3$ $x=\pm 8$

$y=\frac{24}{3}=8$

$y=\frac{24}{-3}=-8$

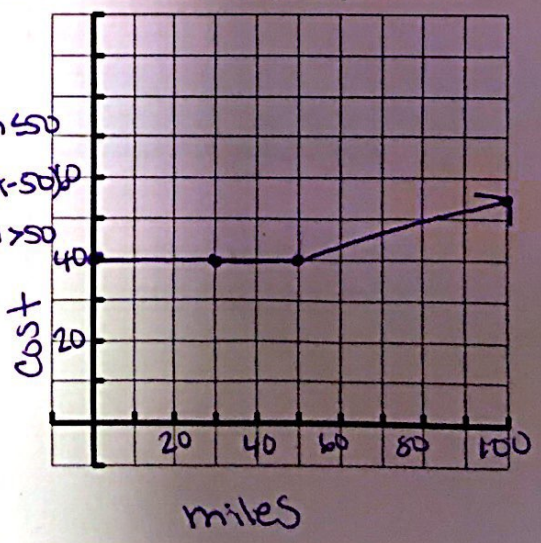
Answer the following appropriately for each application.

18) A rental car company charges a flat fee of \$40 for the first 50 miles drive and an additional \$0.25 per mile for each mile driven in excess of 50 miles.

a) Express the cost, *C*, as a function of the miles driven, *m*.

b) Sketch and label the graph of $C(m)$.

$C(m)=\begin{cases}40 & 0\le m\le 50\\ 40+.25(x-50) & m>50\end{cases}$



c) Determine the cost of driving 200 miles.

$40+.25(200-50)=\$77.50$

d) Determine how many miles one could drive for \$100.

$40+.25(x-50)=100$

$.25(x-50)=60$

$x-50=240$

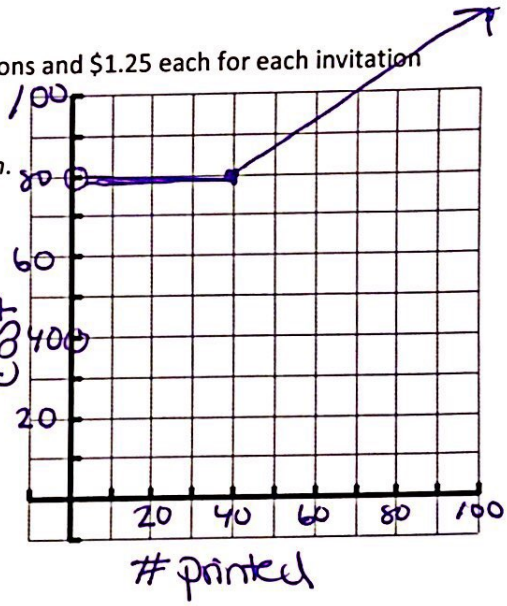
$x=290$ miles

19) Beautiful Wedding charges $2.00 each for printing the first 40 invitations and $1.25 each for each invitation in excess of 40.

a) Express the cost, *C*, as a function of the number printed, *n*.

$C(n) = \begin{cases} 2n & 0 < n \leq 40 \\ 80 + 1.25(n-40) & n > 40 \end{cases}$

b) Sketch and label the graph of *C(n)*.



c) Determine the cost of printing 275 invitations.

$80 + 1.25(275-40) = 373.75$

d) How many invitations can be ordered for a cost of $300?

$80 + 1.25(n-40) = 300$

$1.25(n-40) = 220$

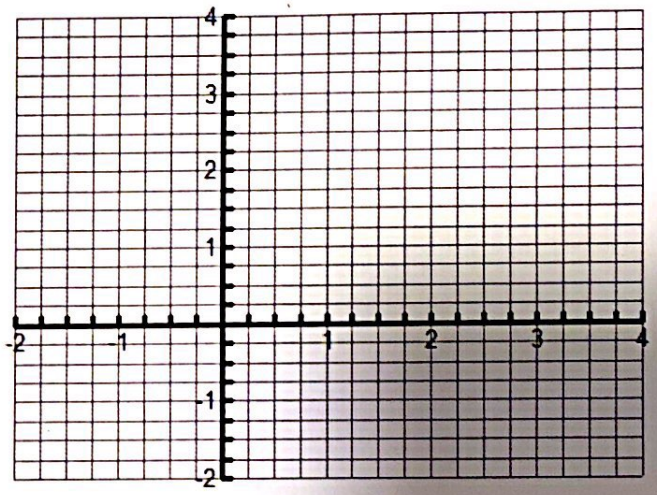
$n - 40 = 176$ 216

Sketch the graph of each function and state the domain and range.

20) $f(x) = [x] + 1$ Omit

Domain: _____

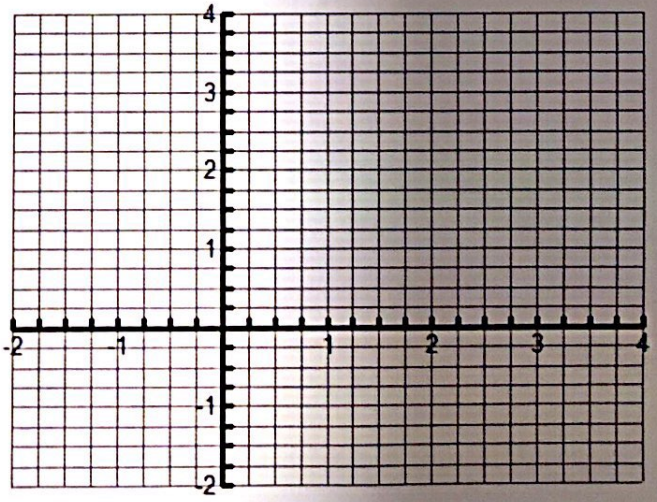
Range: _____



21) $f(x) = [2x]$ Omit

Domain: _____

Range: _____



11) $2x^2+y^2=18$

$xy=4 \longrightarrow y=\frac{4}{x}$

$2x^2+\left(\frac{4}{x}\right)^2=18$

$\cdot x^2$ $2x^2+\frac{16}{x^2}=18$ $\cdot x^2$

$2x^4+16=18x^2$

$2x^4-18x^2+16=0$

$2(x^4-9x+8)=0$

$2(x^2-8)(x^2-1)=0$

$x^2-8=0$ $\quad x^2-1=0$

$x^2=8$ $\quad x^2=1$

$x=\pm\sqrt{8}$ $\quad x=\pm 1$

$x=1$ $\quad x=-1$ $\quad x=\sqrt{8}\approx 2.8$ $\quad x=-\sqrt{8}\approx 2.8$

$y=\frac{4}{1}$ $\quad y=\frac{4}{-1}$ $\quad y=\frac{4}{\sqrt{8}}$ $\quad y=\frac{4}{-\sqrt{8}}$

$y=4$ $\quad y=-4$ $\quad y=1.4$ $\quad y=-1.4$

$(1,4)$ $\quad (-1,-4)$ $\quad (2.8,1.4)$ $\quad (-2.8,-1.4)$

12) $x^2+y^2=4$

$y-x^2=-9 \rightarrow -x^2=-y-9$

$x^2=y+9$

$(y+9)+y^2=4$

$y^2+y+9=4$

$y^2+y+5=0$

$$\frac{-1\pm\sqrt{1^2-4(1)(5)}}{2(1)}$$

$$\frac{-1\pm\sqrt{-19}}{2}$$ ← no solution

can't have a negative under the radical.

13) $x^2 - y^2 = 21$

$x + y = 7 \rightarrow x = 7 - y$

$(7-y)^2 - y^2 = 21$

$49 - 14y + y^2 - y^2 = 21$

$49 - 14y = 21$

$-14y = -28$

$y = 2$

$(5, 2)$

$x = 7 - y$

$x = 5$

14) $x^2+y^2=29$

$xy=10 \rightarrow y=\frac{10}{x}$

$x^2+\left(\frac{10}{x}\right)^2=29$

$\cdot x^2 \quad x^2+\frac{100}{x^2}=29 \quad \cdot x^2$

$x^4+100=29x^2$

$x^4-29x^2+100=0$

$(x^2-4)(x^2-25)=0$

$x^2-4=0$ $\quad$ $x^2-25=0$

$x^2=4$ $\quad$ $x^2=25$

$x=\pm 2$ $\quad$ $x=\pm 5$

$x=2$ $\quad$ $x=-2$ $\quad$ $x=5$ $\quad$ $x=-5$

$y=\frac{10}{2}$ $\quad$ $y=\frac{10}{-2}$ $\quad$ $y=\frac{10}{5}$ $\quad$ $y=\frac{10}{-5}$

$=5$ $\quad$ $=-5$ $\quad$ $=2$ $\quad$ $=-2$

$(2,5)$ $\quad$ $(-2,-5)$ $\quad$ $(5,2)$ $\quad$ $(-5,-2)$