Linear Equations

Write the explicit equation for each of the following tables.

1)

x	f(x)
1	3
2	5
3	7
4	9

Explicit Equation: $f(x) = 3 + 2(x-1)$

2)

x	f(x)
0	3
1	5
2	7
3	9

Explicit Equation: $f(x) = 3 + 2x$

3)

x	f(x)
-2	3
-1	5
0	7
1	9

Explicit Equation: $f(x) = 3 + 2(x+2)$

4) With respect to an arithmetic sequence, what does the change represent? _the common difference_

5) With respect to a linear function, what does the change represent? _the slope_

6)

x	f(x)
3	10
4	14
5	18
6	22

Explicit Equation: $f(x) = 10 + 4(x-3)$

7)

x	f(x)
-4	13
-3	10
-2	7
-1	4

Explicit Equation: $f(x) = 13 - 3(x+4)$

8)

x	f(x)
-4	-10
-2	-7
0	-4
2	-1

Explicit Equation: $f(x) = -10 + \frac{3}{2}(x+4)$

Write the explicit equation for each of the following pairs of points.

1) $(1,5), (2,7)$

x	y
1	5
2	7

Explicit Equation: $y = 5 + 2(x-1)$

2) $(0,2), (1,8)$

x	y
0	2
1	8
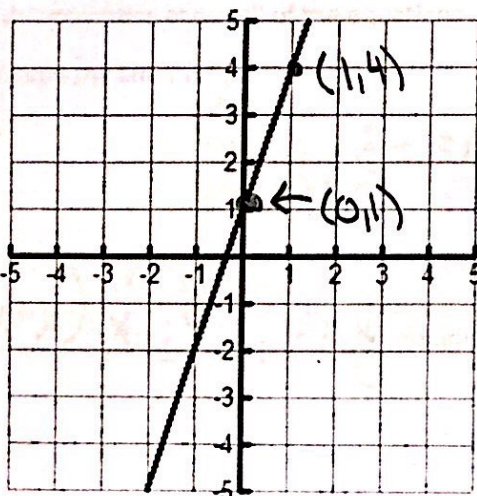
Explicit Equation: $y = 2 + 6x$

3) $(3,1), (6,7)$

x	y
3	1
6	7
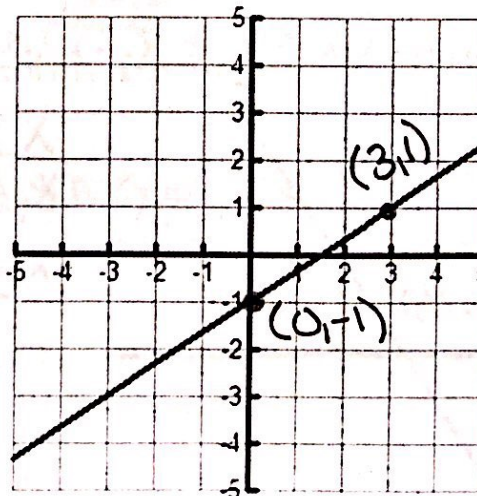
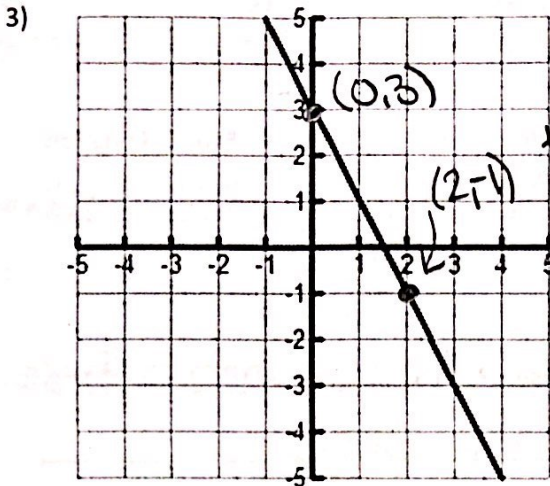Explicit Equation: $y = 1 + 2(x-3)$

4) With respect to an arithmetic sequence, what does the change represent? _the common difference_

5) With respect to a linear function, what does the change represent? _the slope_

Write the explicit equation for each of the following graphs. (Hint: Make a table)

1)

x	y
0	1
1	4

Explicit Equation: $y = 1 + 3x$

2)

x	y
0	-1
3	1

Explicit Equation: $y = -1 + \frac{2}{3}x$

3)

$(0,3)$
$(2,-1)$

$$\begin{array}{c|c} x & y \\ \hline 0 & 3 \\ 2 & -1 \end{array}$$

+2 ↓ ↓ -4

$$d = \frac{-4}{2} = -2$$

Explicit Equation: $y = 3 - 2x$

4)

$(-2,0)$
$(0,-2)$

$$\begin{array}{c|c} x & y \\ \hline -2 & 0 \\ 0 & -2 \end{array}$$
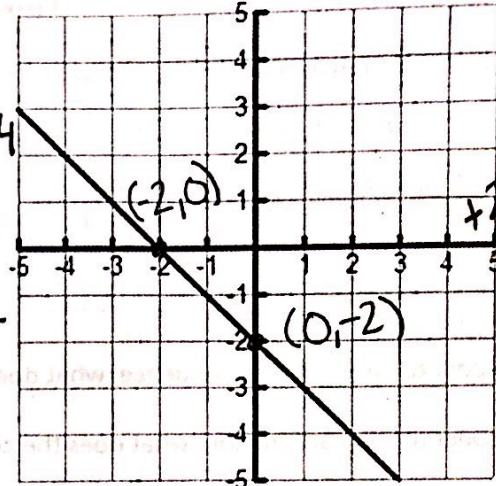
+2 ↓ ↓ -2

$$d = \frac{-2}{2} = -1$$

Explicit Equation: $y = -2 - 1x$

5) What relationships do you see between the tables, pairs of points, and lines?

They are all different ways to express the same thing.

6) What relationships do you see between arithmetic sequences and linear functions.

The x-value is the term # and the y-value is the term.

Each of the following represents a linear function. Find the rate of change and then write explicit equation to each.

1)

$x_2 - x_1$ ↓

x	$f(x)$
x_1	y_1
x_2	y_2
x_3	y_3
x_4	y_1

↓ $y_2 - y_1$

$$d = \frac{y_2 - y_1}{x_2 - x_1}$$

Explicit Equation: $f(x) = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

2) $(x_1, y_1)\ (x_2, y_2)$

$x_2 - x_1$ ↓ $\begin{array}{c|c} x & y \\ \hline x_1 & y_1 \\ x_2 & y_2 \end{array}$ ↓ $y_2 - y_1$

$$d = \frac{y_2 - y_1}{x_2 - x_1}$$

Explicit Equation: $y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

3)

(x_1, y_1)
(x_2, y_2)

$$\begin{array}{c|c} x & y \\ \hline x_1 & y_1 \\ x_2 & y_2 \end{array}$$
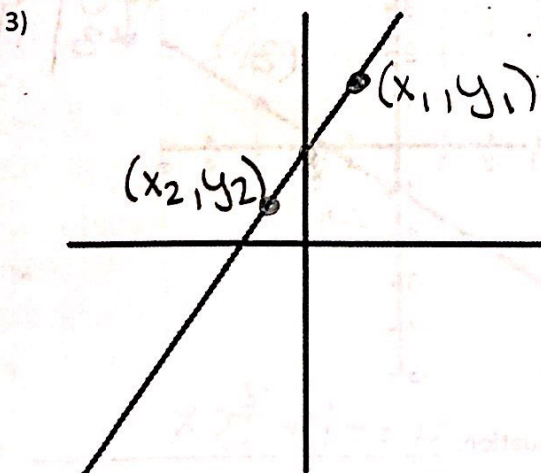
$$d = \frac{y_2 - y_1}{x_2 - x_1}$$

Explicit Equation: $y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

The explicit equations that you found above are all linear. Should have looked like this:

$$f(x) = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x_2 - x_1)$$

Slope formula! ☺

Which is:

$$f(x) = y_1 + m(x_2 - x_1) \quad \text{or} \quad y = y_1 + m(x - x_1)$$

You created the formula for the equation of a linear function. We call this form of a linear function point slope form.

Point Slope Form: $y = m(x - x_1) + y_1$

Using the given information to write the equation of the linear function in point slope form.

1) $m = 2$ and $(2,5)$

$$y = 5 + 2(x-2)$$

2) $m = 2$ and $(4,9)$

$$y = 9 + 2(x-4)$$

3) $(2,5)$ and $(4,9)$

$$y = 5 + 2(x-2)$$
or
$$y = 9 + 2(x-2)$$

x	y
2	5
4	9

$+2 \quad +4$

$$d = m = \frac{4}{2} = 2$$

Simplify each of the equation above by distributing the slope and combining like terms.

Equation #1:

$$y = 5 + 2(x-2)$$
$$y = 5 + 2x - 4$$
$$y = 2x + 1$$

2) Equation #2
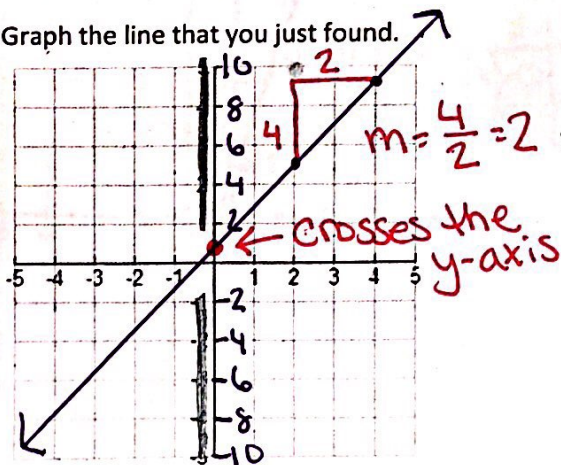
$$y = 9 + 2(x-4)$$
$$y = 9 + 2x - 8$$
$$y = 2x + 1$$

3) Equation #3

Same as equations #1 + #2

What do you notice about all of the equations above?

4) Graph the line that you just found.

$$m = \frac{4}{2} = 2$$

← crosses the y-axis

How do the numbers in the equation that you found relate to the properties of your graph?

✳ Tell students to rescale the y-axis and count by 2.

$$y = 2x + 1$$
↑ m
↑ where the line crosses the y-axis

The numbers in the equation that you found represent the slope and location where the line crosses the y-axis. The number in front of the x represents the slope and the constant term represents the where the line crosses the y-axis.
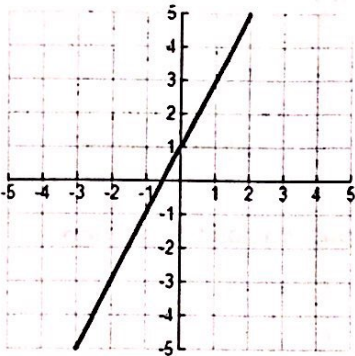
y-intercept – the location where a line crosses the y-axis.
– the y-intercept is represented with the variable b.

Equations written in this form are written in slope intercept form. This form requires both a slope and a y-intercept.

Slope Intercept Form: $y = mx + b$ ← y-intercept
                            slope ↗

Given the slope and y-intercept, or two points, write the equation of each line in slope intercept form.

1) $m = -3$ $b = 9$

$$y = -3x + 9$$

2) $m = \dfrac{5}{3}$ $b = -4$

$$y = \dfrac{5}{3}x + (-4)$$
or
$$y = \dfrac{5}{3}x - 4$$

3) $(-2, 5)$ and $(4, 8)$

$$\begin{array}{c|c} x & y \\ \hline -2 & 5 \\ 4 & 8 \end{array}$$

$$m = \dfrac{3}{6} = \dfrac{1}{2}$$

Homework: Page 44 all and Page 45 all