Rational Functions

Rational Function - A function that can be written as a ratio of two polynomials.

$$f(x) = \frac{p(x)}{q(x)} \quad \text{where } q(x) \neq 0$$

Ex: $f(x) = \frac{x}{x^2-4}$

Domain: The domain of a rational function is all real numbers where $q(x)$ is not equal to zero.

Ex: $(-\infty,-2) \cup (-2,2) \cup (2,+\infty)$

Vertical Asymptotes: Vertical lines that correspond to the zeros of the denominator of a rational function.

Ex: $x=-2$ and $x=+2$

Horizontal Asymptotes: A line that a graph approaches as $x \to \pm\infty$

Ex: $y=0$ * end behavior of the graph

degree numerator < degree denominator $y=0$

Ex: $f(x) = \frac{2x^1+3}{x^2-9}$ $1<2$ HA: $y=0$

degree numerator > degree denominator None

Ex: $f(x) = \frac{x^3-7x}{2x^1+3}$ $3>1$ HA: None

degree numerator = degree denominator $y=$ the ratio of the leading coefficients

Ex: $f(x) = \frac{2x^2-3x+4}{3x^2-7}$ $2=2$

HA: $y=\frac{2}{3}$

Holes: Occur when factors occur in the numerator and denominator.

$$f(x) = \frac{x\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+1)}$$

Hole @ $x=3$

$$f(3) = \frac{x(x+2)}{x+1}$$

$$= \frac{3(3+2)}{3+1}$$

$$= \frac{15}{4}$$

$(3, \frac{15}{4})$

X-intercepts: Occur where $y=0$

y-intercept: Occurs where $x=0$

10) $f(x) = \dfrac{2x+3}{2x^2+5x-3} \rightarrow \dfrac{2x+3}{(2x-1)(x+3)}$

Domain: $(-\infty,-3) \cup (-3,\frac{1}{2}) \cup (\frac{1}{2},+\infty)$

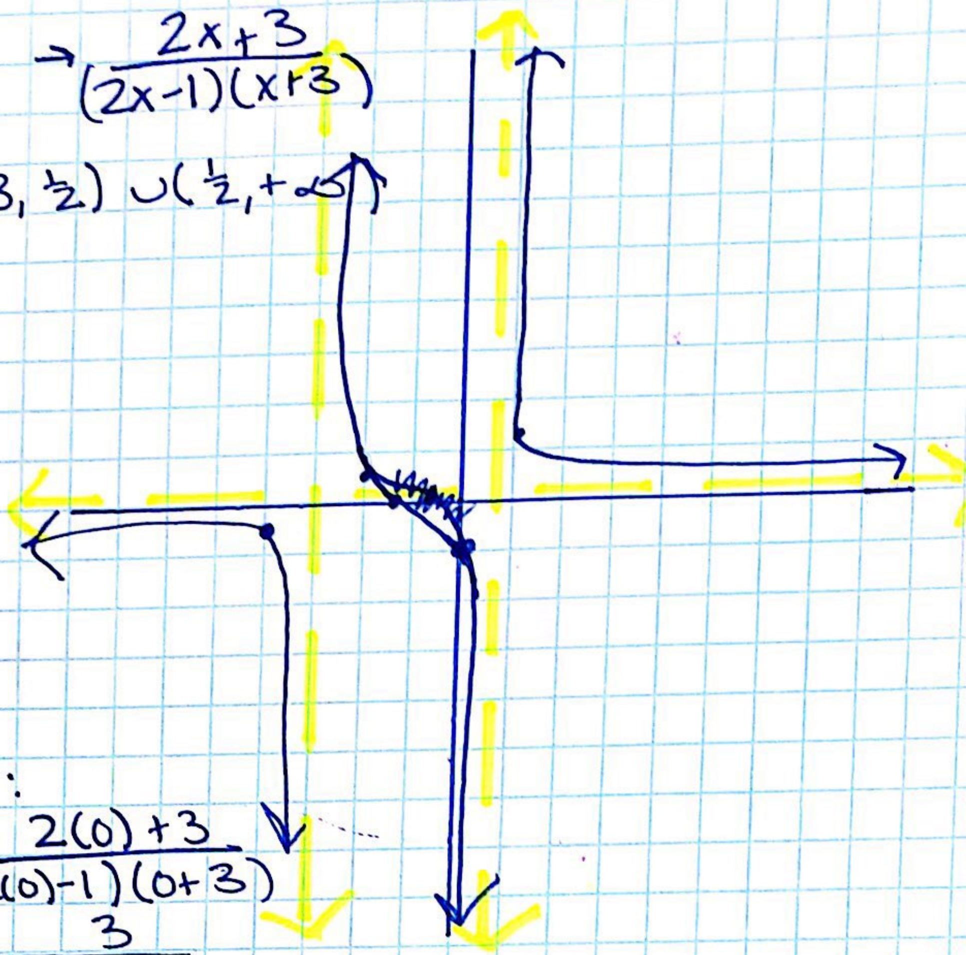
VA: $x=-3$ $x=\frac{1}{2}$

HA: $y=0$

Holes: None

x-int: $(-\frac{3}{2},0)$

y-int: $(0,-1)$



x-int:
$2x+3=0$
$x=\frac{-3}{2}$

y-int:
$y=\dfrac{2(0)+3}{(2(0)-1)(0+3)}$
$=\dfrac{3}{(-1)(3)}$
$=-1$

$x=-2$
$\dfrac{2(-2)+3}{(2(-2)-1)(-2+3)}$

$\dfrac{-1}{(-5)(1)}$

$\dfrac{1}{5}$

$x=1$
$\dfrac{5}{(1)(4)}$

$\dfrac{5}{4}$
$(1,\frac{5}{4})$

$x=-4$
$\dfrac{-5}{(-9)(-1)}$

$\dfrac{-5}{9}$
$(-4,\frac{-5}{9})$

2) $f(x) = \dfrac{3x-6}{x^2-4x+4}$ $\qquad \dfrac{3\cancel{(x-2)}}{(x-2)\cancel{(x-2)}} = \dfrac{3}{x-2}$

Domain: $(-\infty, 2) \cup (2, +\infty)$

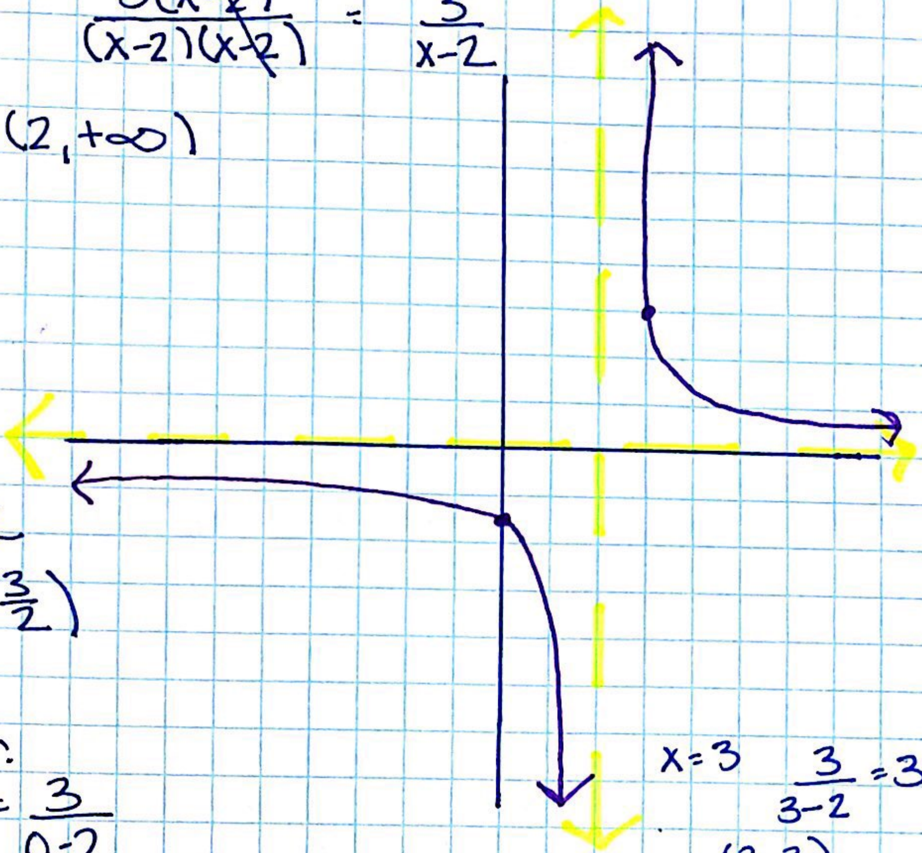
VA: $x = 2$

HA: $y = 0$

Holes: None

x-intercepts: None

y-intercepts: $(0, -\frac{3}{2})$



$x = 3$ $\quad \dfrac{3}{3-2} = 3$

$(3, 3)$

x-int:

$0 = \dfrac{3}{x-2}$

y-int:

$y = \dfrac{3}{0-2}$

$y = -\dfrac{3}{2}$

8) $g(x) = \dfrac{2x-5}{6x^2-13x-5} \rightarrow \dfrac{\cancel{2x-5}}{(3x+1)\cancel{(2x-5)}} \rightarrow \dfrac{1}{3x+1}$

Domain: $(-\infty, -\frac{1}{3}) \cup (-\frac{1}{3}, \frac{5}{2}) \cup (\frac{5}{2}, +\infty)$

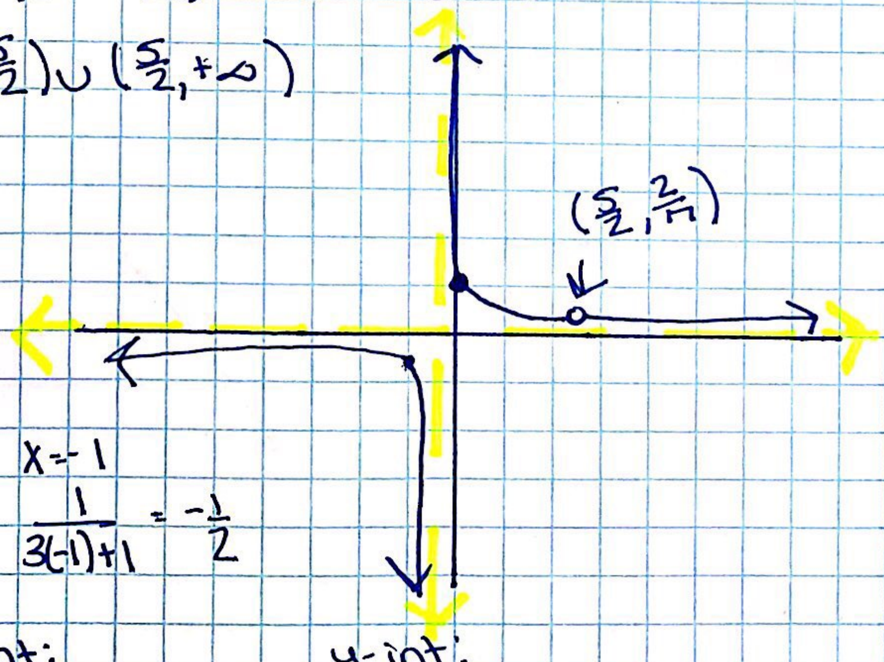
VA: $x = -\frac{1}{3}$

HA: $y = 0$

Holes: $(\frac{5}{2}, \frac{2}{17})$

x-int: None

y-int: $(0, 1)$



$x = -1$

$\dfrac{1}{3(-1)+1} = -\dfrac{1}{2}$

Hole: $\dfrac{1}{3(\frac{5}{2})+1}$

$\dfrac{1}{\frac{15}{2}+1} = \dfrac{2}{17}$

x-int:

$0 = \dfrac{1}{3x+1}$

y-int:

$y = \dfrac{1}{3(0)+1}$

$y = 1$

6) $f(x) = \dfrac{3x^2+16x+5}{x^2-3x-10} \rightarrow \dfrac{(3x+1)(x+5)}{(x-5)(x+2)}$

Domain: $(-\infty,-2)\cup(-2,5)\cup(5,+\infty)$

VA: $x=5 \quad x=-2$

HA: $y=3$

Holes: None

x-int: $(-\frac{1}{3},0)\ (-5,0)$

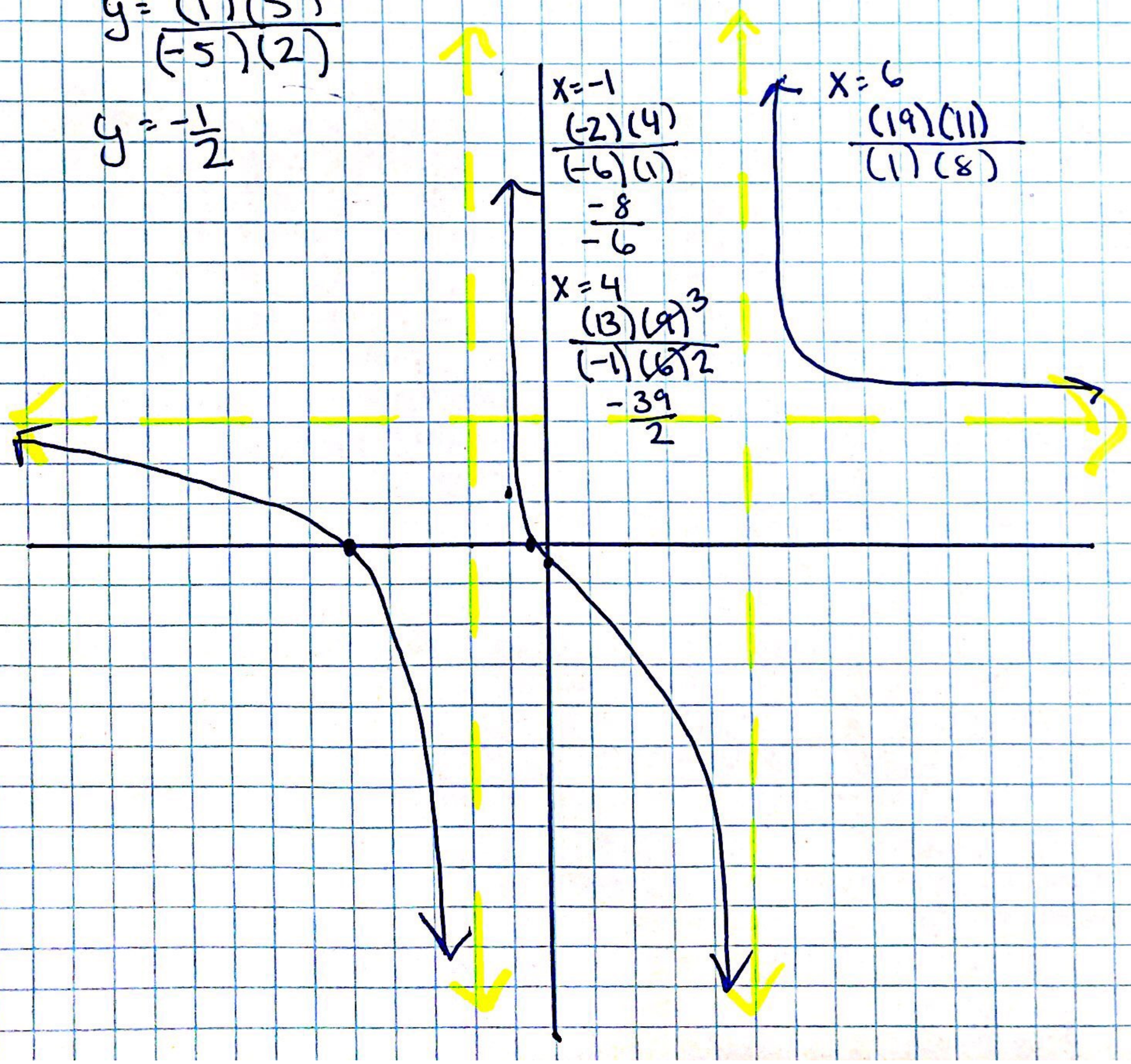
$(3x+1)(x+5)=0$

$x=-\frac{1}{3} \quad x=-5$

y-int: $(0,-\frac{1}{2})$

$y=\dfrac{(1)(5)}{(-5)(2)}$

$y=-\dfrac{1}{2}$



$x=-1$

$\dfrac{(-2)(4)}{(-6)(1)}$

$\dfrac{-8}{-6}$

$x=4$

$\dfrac{(13)(\cancel{9})^3}{(-1)(\cancel{6})2}$

$-\dfrac{39}{2}$

$x=6$

$\dfrac{(19)(11)}{(1)(8)}$