Proof by Induction

3 Steps:

1) Prove that it works for $n = 1$
2) Assume that the sequence holds true for a finite # of terms

   Assume it works for $n = k$
3) Prove that it works for $n = k+1$

1) Prove: $1 + 3 + 5 + \ldots + (2n-1) = n^2$

Step 1: Show it holds true for $n = 1$

Let $n = 1$

$$(2(1) - 1) = 1^2$$
$$1 = 1$$

Step 2: Assume it holds true for $n = k$

Assume $n = k$

$$1 + 3 + 5 + \ldots + (2k-1) = k^2$$

Step 3: Prove for $n = k+1$

$$\underbrace{1 + 3 + 5 + \ldots + (2k-1)}_{\text{Assumption}} + [2(k+1) - 1] = (k+1)^2$$

$$k^2 + (2k + 2 - 1) = (k+1)^2$$
$$k^2 + 2k + 1 = (k+1)^2$$
$$(k+1)^2 = (k+1)^2$$

2) Prove: $2+6+10+\ldots+\overbrace{(4n-2)}^{a_n}=2n^2$

Step1: Let $n=1$

$$4(1)-2=2(1)^2$$
$$2=2$$

Step2: Assume $n=K$, thus the induction hypothesis is

$$\underline{2+6+10+\ldots+(4K-2)}=2K^2$$

Step3: Prove true for $n=K+1$

$$\underline{2+6+10+\ldots+(4K-2)}+[4(K+1)-2]=2(K+1)^2$$

$$2K^2+[4(K+1)-2]=2(K+1)^2$$
$$2K^2+(4K+4-2)=2(K+1)^2$$
$$2K^2+4K+2=2(K+1)^2$$
$$2(K+2K+1)=2(K+1)^2$$
$$2(K+1)^2=2(K+1)^2$$