

Logarithmic Functions

Common Log - The common logarithm of any positive real number x is defined to be the exponent you get when you write x as a power of 10.

$$\log_{10} x = a \text{ if and only if } 10^a = x$$

Logs of Different Bases

$$\log_b x = a \text{ then } b^a = x$$

base \rightarrow b \rightarrow x \rightarrow a \rightarrow $b^a = x$

Natural Log - log base e

$$\log_e x = a \rightarrow \ln x = a$$

Logarithm \rightarrow Exponential

$$1) \log_3 81 = 4 \quad 3^4 = 81 \quad 2) \log_2 \frac{1}{8} = -3 \quad 2^{-3} = \frac{1}{8}$$

Exponential \rightarrow Logarithm

$$1) 3^5 = 243 \quad \log_3 243 = 5 \quad 2) 4^{-2} = \frac{1}{16} \quad \log_4 \frac{1}{16} = -2$$

Evaluating Logarithmic Expressions

$$1) \log_{49} 7 = x$$

$$49^x = 7$$

$$7^{2x} = 7$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$2) \log_9 \sqrt{3} = x$$

$$9^x = \sqrt{3}$$

$$3^{2x} = 3^{\frac{1}{2}}$$

$$2x = \frac{1}{2}$$

$$x = \frac{1}{4}$$

$$3) \log_3 3^2 = x$$

$$3^x = 3^2$$

$$x = 2$$

$$\log_{49} 7 = \frac{1}{2}$$

$$\log_9 \sqrt{3} = \frac{1}{4}$$

$$\log_3 3^2 = 2$$

$$4) \ln e^4 = x$$

$$e^x = e^4$$

$$x = 4$$

$$5) \ln\left(\frac{1}{e}\right)$$

$$\ln e^{-1} = x$$

$$e^x = e^{-1}$$

$$6) \log_5 21 = x$$

$$\log_5 x = \log_5 21$$

$$x = 21$$

$$\ln e^4 = 4$$

$$\ln\left(\frac{1}{e}\right) = -1$$

$$\log_5 5^{\log_5 21} = 21$$