13.5 Sums of Infinite Series

If $|r|<1$, the infinite geometric series converges to a sum.

$$S = \frac{a_1}{1-r}$$

If $|r| \geq$, then the infinite geometric series diverges.

Ex: Find each sum

1) $1+2+3+4+\ldots$ Divergent

2) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots$ Convergent $r=\frac{1}{2}$ $S=\frac{\frac{1}{2}}{1-\frac{1}{2}}=1$

3) $\frac{1}{27}+\frac{1}{9}+\frac{1}{3}+1+\ldots$ Divergent $r=3$

4) $1+.1+.01+.001+.0001+\ldots$ Convergent $r=\frac{1}{10}$ $S=\frac{10}{9}$

Ex: For what values of x does the following series converge?

$$1+(x-3)+(x-3)^2+(x-3)^3+\ldots$$

$r = x-3$ $\quad |x-3|<1$

~~$x<3$~~

$x-3<1$ and $x-3>-1$

$x<4$ $\quad x>2$

Interval of Convergence $\quad 2<x<4$