



Modesto City Schools

MVP Math 1: Module 2

Extra Help & Examples

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2.1 Ready, Set, Go!

Ready

Topic: Recognizing arithmetic and geometric sequences

To determine if the sequence is arithmetic or geometric you will need to see if you have a constant difference or common ratio. An arithmetic sequence will have a pattern that adds (or subtracts) the same number each time. A geometric sequence will have a pattern that multiplies (or divides) by the same number each time.

Example 1: For the given sequence, find the missing values. Underline if it has a constant difference or common ratio. State the value of the difference or ratio. Finally, indicate if the sequence is arithmetic or geometric by circling the answer.

Sequence: 15, 12, ____, 6, 3, ____

Missing terms: 15, 12, 9, 6, 3, 0

Constant difference or ratio?

Constant difference/ratio = -3

Arithmetic or Geometric?

Before we can find the missing terms, we need to see if we are adding or multiplying. We have 2 pairs of consecutive terms, 15, 12 and 6, 3 so we use these terms to see if we have a constant difference. First we subtract (working backwards) $12 - 15 = -3$. We want to see if the next pair of consecutive terms also have the same difference: $3 - 6 = -3$. This means that we have a constant difference of -3. A sequence with a constant difference is arithmetic. Finally, we can use the pattern of subtracting 3 to find the missing terms.

Example 2: For the given sequence, find the missing values. Underline if it has a constant difference or common ratio. State the value of the difference or ratio. Finally, indicate if the sequence is arithmetic or geometric by circling the answer.

Sequence: 1, 3, ____, 27, 81, ____

Missing terms: 1, 3, 9, 27, 81, 243

Constant difference or ratio?

Constant difference/ratio = 3

Arithmetic or Geometric?

We begin by checking to see if we have a constant difference: $3 - 1 = 2$ and $81 - 27 = 54$. The answers are not the same, so we do not have a constant difference. Next, we check to see if we have a common ratio: $\frac{3}{1} = 3$ and $\frac{81}{27} = 3$. The ratios are the same, so we have a common ratio of 3. A sequence with a common ratio is geometric. Finally, we can use the pattern of multiplying by 3 to find the missing terms.

Video Examples: <https://youtu.be/ri767qbKLC4>

Set

Topic: Discrete and continuous relationships

Discrete data can only take on certain values. For example, the number of students in your math class is discrete because you can only use whole numbers (you can't have half a student). Discrete data is countable. Here are some more examples of discrete data:

Ex. The results of rolling 2 dice. You can only get the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 as results.

Ex. The number of pennies in a piggy jar. You can count the number of pennies and you can't have a fraction of a penny.

Ex. The number of books in your backpack. You can count the number of books.

Continuous data can take on any value within the domain. For example, the heights of the people in your math class can take on any values within normal human heights (you can be 5.5 ft tall). Continuous data is measurable. Here are some more examples of continuous data:

Ex. How long it takes a person to complete a race (you can measure time, even fractions of a second).

Ex. The amount of water in gallons in a pool increases over time (you can measure the gallons, even fractions of a gallon).

Ex. The temperature in a car increases over time on a sunny day (you can measure the temperature).

Video help: <https://youtu.be/6ldJ1aPFDCs>

http://viewpure.com/_yAQb8gWBpU?start=0&end=0

Go

Topic: Solving one-step equations

A ratio is a comparison of values. For example, if you have 22 boys in your class and 17 girls, the ratio of boys to girls would be 22:17 or $\frac{22}{17}$. When we compare 2 values that have different units, we are creating a rate. For example, if a Snickers bar costs \$1.29 for 1.86 ounces, we can write the ratios as \$1.29:1.86 oz or $\frac{\$1.29}{1.86oz}$. This means that we will pay \$1.29 for every 1.86 ounces of the Snickers bar. A unit rate is a rate where we compare the value to 1 unit. For example, if we compare the number of miles we drive to one hour, we would have a unit rate of "miles per hour." If I drive 50 miles in one hour, my unit rate is 50 miles per hour. Some common unit rates are "miles per hour", "miles per gallon", "price per pound". Another key word that means you're dealing with a unit rate is "each." So if you see a sign that says apples are \$0.79 each, it means \$0.79 PER apple. To find a unit rate, we divide the first value by the second value.

Ex. In the Snickers example, the price was \$1.29 per 1.86 ounces. If I want to find the unit rate, or "price per ounce", I will divide the price by the number of ounces.

$1.29 \div 1.86 \approx 0.69$ So the price per ounce would be \$0.69.

Ex. If I drive 120 miles in 4 hours, I can find the unit rate, or "miles per hour" by dividing miles by hours.

$120 \div 4 = 60$ So the unit rate would be 60 miles per hour (60 miles in ONE hour)

We can use the unit rate to solve problems. For example, if I know that it costs \$2.30 for one box of cereal, I can find out how much money it will cost to buy 4 boxes of cereal (before tax). We can multiply the unit rate by the quantity to find the total.

Ex. If 1 box of cereal costs \$2.30 and I want to buy 4 boxes of cereal, I will multiply the unit rate by the number of boxes. $\$2.30 \times 4 = \9.20 So the price before tax for 4 boxes of cereal would be \$9.20.

Ex. If 1 pound of apples costs \$0.60 and I want to buy 5 pounds, I can multiply the unit rate by the amount I want. So, $\$0.60 \times 5 = \3.00 means that 5 pounds would cost \$3.00

If I know the unit rate, and I know the total cost, I can find out how many items I purchased by setting up an equation.

Ex. If 1 pound of apples costs \$0.60 and I spend \$6.60, how many pounds of apples did I buy? I know that if I multiply the unit price by the number of pounds, I will get the total cost. I can set up an equation:

$$\boxed{\text{price per pound}} \times \boxed{\text{number of pounds}} = \boxed{\text{total cost}}$$

$0.60x = 6.60$; x represents the number of pounds. I can solve this equation by dividing.

$$\frac{0.60}{0.60}x = \frac{6.60}{0.60}$$

$x = 11$ This means that we purchased 11 pounds of apples.

Video help: http://viewpure.com/liW_ALj4Qj8?start=0&end=0

<https://www.virtualnerd.com/pre-algebra/ratios-proportions/rates-unit-rates/rate-unit-rate-examples/rate-and-unit-rates-definition>

<https://www.virtualnerd.com/pre-algebra/ratios-proportions/rates-unit-rates/rate-unit-rate-examples/rate-to-unit-rate-conversion>

<https://www.virtualnerd.com/pre-algebra/ratios-proportions/rates-unit-rates/rate-unit-rate-examples/rates-word-problem-solution>

<http://viewpure.com/l3XzepN03KQ?start=0&end=0>

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2.2 Ready, Set, Go!

Ready

Topic: Comparing rates of change in linear situations

A unit rate is a rate where we compare the value to 1 unit. For example, if we compare the number of miles we drive to one hour, we would have a unit rate of “miles per hour.” If I drive 50 miles in one hour, my unit rate is 50 miles per hour. Some common unit rates are “miles per hour”, “miles per gallon”, “price per pound”. Another key word that means you’re dealing with a unit rate is “each.” So if you see a sign that says apples are \$0.79 each, it means \$0.79 PER apple. To find a unit rate, we divide the first value by the second value.

When you compare two situations, you can determine which has the greatest rate of change by finding the unit rate. You need to make sure that both cases use the same units before you can compare them. For example, if one unit rate is miles per hour and the other is miles per minute, we would not get an accurate comparison. Once both situations are written using the same units, you can determine which has the greatest unit rate.

Here are some helpful conversions:

1 hour can be written as 60 minutes

1 foot can be written as 12 inches

1 yard can be written as 3 feet

1 day can be written as 24 hours

1 pound is 16 ounces

Ex. A turtle that traveled 3 ft. in 2 hours or an inch worm that travelled 7 in. in 12 minutes.

We need to find the unit rate for both scenarios before we can compare. When we find the unit rate, we want to make sure that we are looking at the same units. The turtle’s speed is measured in feet/hour and the inch worm’s speed is measured in inches/minute. We can convert the units using multiplication:

Turtle:

$$\begin{aligned} \frac{3 \text{ feet}}{2 \text{ hours}} \times \frac{12 \text{ inches}}{1 \text{ foot}} &= \frac{36 \text{ inches}}{2 \text{ hours}} \\ \frac{36 \text{ inches}}{2 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} &= \frac{36 \text{ inches}}{120 \text{ minutes}} \end{aligned}$$

Inch Worm:

$$\frac{7 \text{ inches}}{12 \text{ minutes}}$$

Now that both situations are in inches per minute, we can find the unit rate by dividing:

Turtle:

$$\frac{36 \text{ inches}}{120 \text{ minutes}} = \frac{0.3 \text{ inches}}{1 \text{ minute}}$$

Inch Worm:

$$\frac{7 \text{ inches}}{12 \text{ minutes}} = \frac{0.58\bar{3} \text{ inches}}{1 \text{ minute}}$$

Next, we can compare the unit rates. The inch worm covers more distance in 1 minute, so it has the greater rate of change.

Here is another example:

Ex. 1 pound of frozen yogurt for \$4.95 or a 12 ounce ice cream cone for \$3.75. The frozen yogurt is in terms of pounds and the ice cream is in terms of ounces. We first need to convert the units to price and ounces:

Frozen Yogurt:

$$\frac{\$4.95}{1 \text{ pound}} \times \frac{1 \text{ pound}}{16 \text{ ounces}} = \frac{\$4.95}{16 \text{ ounces}}$$

Ice Cream:

$$\frac{\$3.55}{12 \text{ ounces}}$$

Next, we can find the unit rate by dividing the price by the weight:

Frozen Yogurt:

$$\frac{\$4.95}{16 \text{ ounces}} \approx \frac{\$0.31}{1 \text{ ounce}}$$

Ice Cream:

$$\frac{\$3.55}{12 \text{ ounces}} = \frac{\$0.30}{1 \text{ ounce}}$$

If we want to find the greater rate of change, we look for the bigger unit rate. In this case, the frozen yogurt has the greatest rate of change. This means that the frozen yogurt costs more per ounce.

Video Examples: <http://viewpure.com/YHm5uWik-ZA?start=0&end=0>

<http://viewpure.com/XA5dijonato?start=0&end=0>

<http://viewpure.com/LBGH-0YV-44?start=0&end=0>

Set

Topic: Discrete and continuous relationships

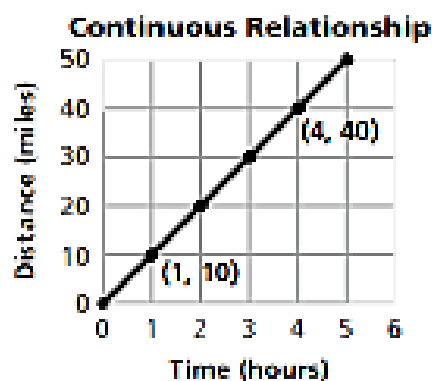
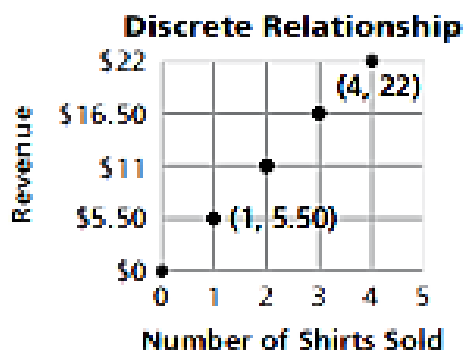
Discrete data can only take on certain values. For example, the number of students in your math class is discrete because you can only use whole numbers (you can't have half a student). Continuous data can take on any value within the domain. For example, the heights of the people in your math class can take on any values within normal human heights (you can be 5.5 ft tall). Continuous data is measurable.

When you are given a situation you should ask if the data can be counted or measured:

Ex. The population of a city each year (population is people). We're counting the number of people, so this is discrete.

Ex. The temperature of a steak increases while it's on the BBQ. The temperature can be measured, so this is continuous.

When you are given a graph, you can look to see if the points on the graph are NOT connected (discrete) or if the points form a connected line or curve (continuous).



When you are given an equation, you can look to see if the equation is recursive (sequences only use whole number inputs). Recursive equations are used for sequences, which are discrete. Explicit functions often describe continuous data.

Discrete Relationship

Continuous Relationship

Ex. Recursive: $f(1) = 2; f(n) = f(n-1) + 3$
 Explicit: $f(n) = 2 + 3(n-1)$

$$f(x) = 3x - 1$$

Sequences use "n" for term number

Functions often use x as the independent variable

Arithmetic sequences have a common difference, so you will add (or subtract) by the same number each time. Geometric sequences have a common ratio, so you will multiply (or divide) by the same amount each time. If the sequence does not have a common difference or common ratio, it is neither arithmetic or geometric.

We can first check if the situation is linear (arithmetic) by looking for a constant difference. If the situation involves increasing or decreasing by the same amount each time, it is linear. If the context requires multiplying (or dividing) by a constant factor each time, it is exponential (geometric).

Ex. Each week you earn \$5 by doing chores around the house. In this case, you are adding \$5 each week. The rate of change is constant so the context is linear (arithmetic).

Ex. Each month you earn 3% interest on your savings. We calculate percent by multiplying, so this is an exponential (geometric) context.

***In a context, you can look for key terms that will help you identify if the situation can be modeled by a linear or exponential equation.

Linear: Miles per hour, feet per second, price for each item, etc.

Exponential: Increase by a percent (%), doubles, triples, half as much, etc.

If you are trying to determine if a situation is linear or exponential from a recursive equation, you will look to see if you are adding/subtracting a constant each time or multiplying/dividing each time.

Linear (arithmetic)

Exponential (geometric)

Recursive: $f(1) = 2; f(n) = f(n-1) + 3$

Recursive: $f(1) = 2; f(n) = f(n-1) \times 3$

If you are trying to determine if a situation is linear or exponential from an explicit equation, you will look to see if you have a slope or if you have a variable as an exponent.

Linear (arithmetic)

Exponential (geometric)

Explicit: $f(x) = 3x - 1$

Explicit: $f(x) = 2 \times 3^{x-1}$

Video help:

<https://video.search.yahoo.com/search/video?fr=tightropetb&p=discrete+vs+continuous#id=1&vid=9e099ce4a58806b5f26d155f1d2a1cf3&action=click>

<https://youtu.be/6ldJ1aPFDCs>

http://viewpure.com/_yAQb8gWBpU?start=0&end=0

https://learnzillion.com/lesson_plans/6415

https://learnzillion.com/lesson_plans/6663-distinguish-between-linear-and-exponential-functions-using-tables

<http://viewpure.com/EITZBgr9xR8?start=0&end=0>

<http://viewpure.com/fw5ekx-C-2A?start=0&end=0>

Go

Topic: Solving one-step equations

The goal of solving an equation is to determine the value of the variable. When solving a one-step equation, our job is to use inverse (opposite) operations to get the variable by itself. Why is this the case?

Let's look at an example:



The weight of the items can be modeled by the equation: shoes + socks = 13.9 ounces. If we know that the socks weigh 0.7 ounces, we can figure out how much the shoes weigh. We know that the combined weight is 13.9 ounces and the socks take up 0.7 ounces of the total.

$$\text{Shoes} + 0.7 \text{ ounces} = 13.9 \text{ ounces}$$

We know that the total is 13.9 ounces and we've used up 0.7 ounces. The weight of the shoes will be the amount of ounces left after we take away the weight of the socks.

$$\text{Shoes} = 13.9 \text{ ounces} - 0.7 \text{ ounces}$$

$$\text{Shoes} = 13.2 \text{ ounces}$$

Our original equation was:

$$\text{shoes} + \text{socks} = 13.9 \text{ ounces}$$

We were told that the socks weigh 0.7 ounces:

$$\text{shoes} + 0.7 = 13.9 \text{ ounces}$$

We subtracted 0.7 to find the weight of the shoes:

$$\begin{array}{r} -0.7 \quad -0.7 \\ \text{shoes} + 0.7 = 13.9 \\ \hline \text{shoes} = 13.2 \end{array}$$

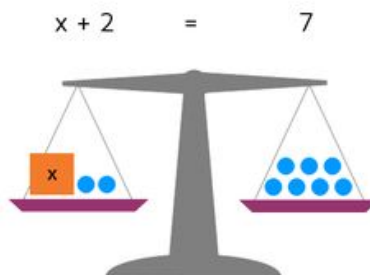
The shoes weighed 13.9 - 0.7 ounces:

$$\text{shoes} = 13.2 \text{ ounces}$$

Our original equation had addition, but we had to use the inverse operation (subtraction) to solve.

Here is another example:

In this picture, the orange block represents x , and the blue circles each represent 1 unit. So on the left, 2 circles are being added to x and on the right we have 7 circles. The two sides balance.



To find the weight of the orange block, we need to remove (subtract) the 2 circles. We have to be careful though, if we only take 2 circles from the left side, the scale won't balance.

We can solve this equation algebraically:

$$x + 2 = 7$$

We need to take away 2 circles from each side:

$$\begin{array}{r} -2 \quad -2 \\ x + 2 = 7 \\ \hline x = 5 \end{array}$$

If we remove 2 blocks, the orange block is equal to 5 circles:

$$x = 5$$

This process of using the inverse (opposite) operation to solve an equation will work with operations other than adding. Here are some more examples:

$y - 3 = -5$	Original Problem
$y - 3 = -5$	We want to remove the minus 3.
$y - 3 + 3 = -5 + 3$	The opposite of minus 3 is plus 3, so I added 3 to BOTH sides of the equation.
$y = -2$	$-3 + 3 = 0$, so y remains on the left and $-5 + 3 = -2$; therefore $y = -2$
Check: $y - 3 = -5$ $-2 - 3 = -5$ $-2 + (-3) = -5$	Since this is a correct statement, my answer of $y = -2$ is also correct.

$-2y = 18$	Original Problem
$-2y = 18$	-2 is the coefficient that we need to remove by division!
$\frac{-2y}{-2} = \frac{18}{-2}$	Divide BOTH sides of the equation by -2 .
$y = -9$	$\frac{18}{-2} = -9$, so $y = -9$
Check: $-2y = 18$ $-2(-9) = 18$	Since this statement is correct, my answer $y = -9$ is correct!

$\frac{x}{2} = 14$	Original Problem
$\frac{x}{2} = 14$	We want to remove the denominator of 2, so we are going to multiply by 2.
$2 \cdot \frac{x}{2} = 14 \cdot 2$	Multiply by 2 on BOTH sides of the equation!
$x = 28$	$14 \cdot 2 = 28$
Check: $\frac{x}{2} = 14$ $\frac{28}{2} = 14$	28 divided by 2 is a true statement, so my answer $x = 28$ is correct!

$\frac{-3}{5}x = 2$	Original Problem
$\frac{-3}{5}x = 2$	We want to remove the $-\frac{3}{5}$ by multiplying by the reciprocal.
$\frac{-5}{3} \cdot \frac{-3}{5}x = \frac{2}{1} \cdot \frac{-5}{3}$	Multiply BOTH sides by $-\frac{5}{3}$
$x = \frac{-10}{3}$ or $-3\frac{1}{3}$	$\frac{2}{1} \cdot \frac{-5}{3} = \frac{-10}{3}$
Check: $\frac{-3}{5}x = 2$ $\frac{-3}{5} \cdot \frac{-10}{3} = \frac{30}{15} = 2$	Since this statement is correct, my answer of $x = -10/3$ is also correct!

***For the last example, x is being multiplied by $\frac{-3}{5}$. The opposite of multiplying would be to divide by $\frac{-3}{5}$. However, when we divide with fractions, we flip and multiply.

Video help:

<https://www.khanacademy.org/math/algebra/one-variable-linear-equations#alg1-one-step-add-sub-equations>

<https://www.khanacademy.org/math/algebra/one-variable-linear-equations#alg1-one-step-mult-div-equations>

http://viewpure.com/6Buj_mPTW3o?start=0&end=0

2.3 Ready, Set, Go!

Ready

Topic: Comparing rates of change in both linear and exponential situations

Linear functions have a constant rate of change (slope). When you have different representation, you will need to use different methods to find the rate of change.

Example 1 Find the Average Rate of Change from a Table

The table below defines the relationship $y = f(x)$.

x	0	2	4	5
$f(x)$	26	17	5	1

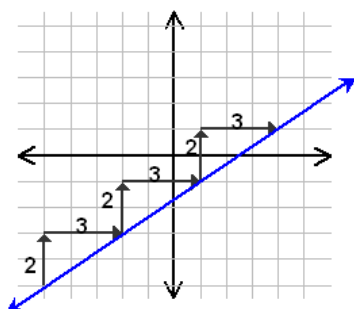
Use this table to compute the average rates of change below.

a. Find the average rate of change of f with respect to x over $[0,4]$.

Solution Apply the definition of the average rate of change to give

$$\begin{aligned} \text{Average rate of change of } f &= \frac{\text{Change in } f}{\text{Change in } x} \\ \text{with respect to } x \text{ over } [0,4] &= \frac{f(4) - f(0)}{4 - 0} && \text{Find the change in the numerator} \\ &= \frac{5 - 26}{4 - 0} && \text{and denominator over } [0, 4] \\ &= \frac{5 - 26}{4 - 0} && \text{Put in the function values from} \\ &= \frac{-21}{4} && \text{the table} \end{aligned}$$

Example 2 Find the Average Rate of Change from a Graph



$$\text{Rise} = 2$$

$$\text{Run} = 3$$

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{2}{3}$$

Example 3 Find the rate of change from a word problem

- A package of cheddar cheese contains 15 servings and has a total of 147 grams of fat. Find the unit rate of grams of fat per serving.

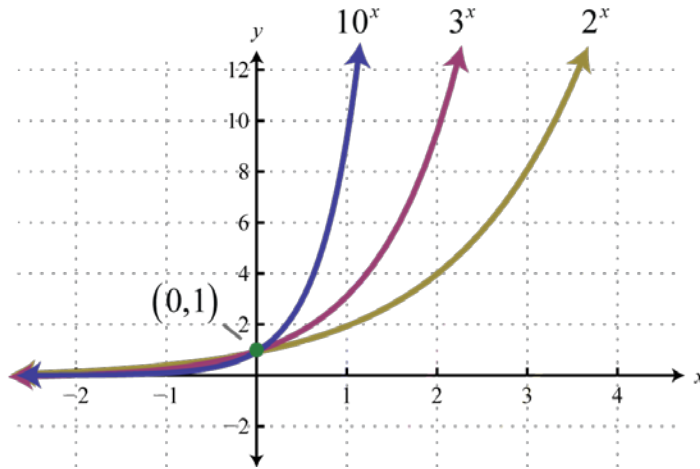
To find the unit rate, divide the numerator and the denominator by the denominator.

$$\frac{147 \text{ grams}}{15 \text{ servings}} \div \frac{15}{15} = \frac{9.8 \text{ grams}}{1 \text{ serving}}$$

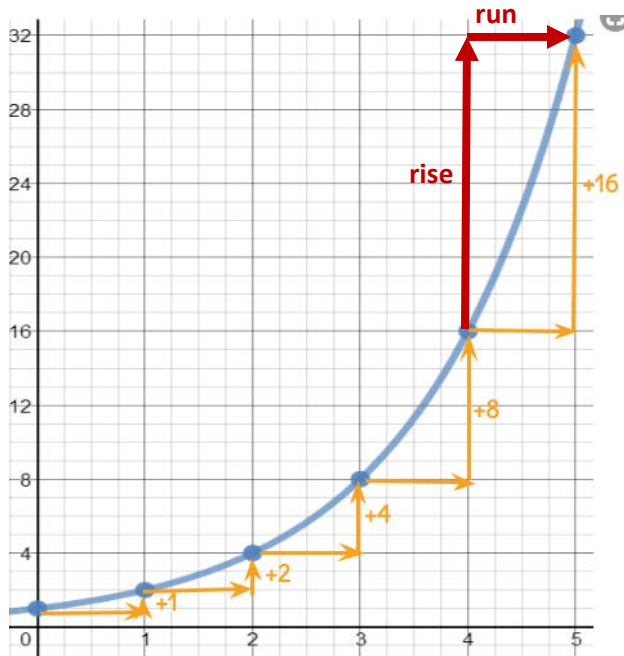
Exponential functions do not have a constant rate of change, instead they have a common ratio.

Example: Given graphs of exponential functions

When you compare graphs of exponential functions, you can compare the “steepness” of the line. The curve that turns upward more sharply is growing faster. In the graph below the function $y = 10^x$ is growing fastest. The graph of $y = 3^x$ is growing slower than $y = 10^x$, but faster than $y = 2^x$. The graph of $y = 2^x$ is growing the slowest.



You can calculate the average rate of change for an interval by drawing a slope triangle from the endpoints of the interval.



If I want to calculate the average rate of change for the function to the left, I will need to determine the interval I want to observe. For example, the average rate of change over the interval $[4,5]$ can be modeled by the triangle to the left. For this interval the rise is 16 and the run is 1. So the average rate of change would be $\frac{16}{1}$ or 16. For a line, the average rate of change stays the same. For an exponential, the average rate of change will be different for different intervals.

Example: Given the explicit equation for an exponential function.

Which function is growing the fastest?

$y = 4^x$ $y = (\frac{1}{2})^x$ $y = 8^x$

How can you tell?

It has the highest base!

Once you have the rate of change, you can compare to see which rate is greater.

Video help: <https://www.khanacademy.org/math/algebra/introduction-to-exponential-functions/exponential-vs-linear-growth/v/exponential-vs-linear-growth>

<http://viewpure.com/GGiiadll1ag?start=0&end=0>

<http://viewpure.com/iwTu4fHoyZ0?start=0&end=0>

Set

Topic: Recognizing linear and exponential functions

Linear functions have several defining characteristics:

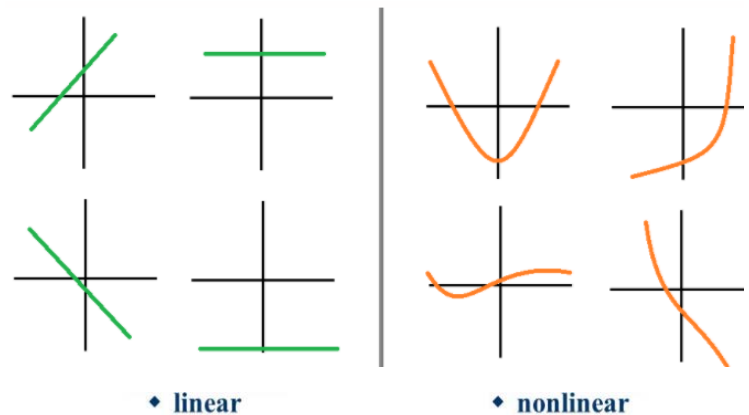
- Their graph will be a straight line (or segment depending on the context).
- They have a constant rate of change (slope/common difference).
- The equation is a first degree equation (x is to the 1st power so we typically don't write the exponent).
 - Standard form: $Ax + By = C$ (A, B, and C are whole numbers and A is positive). Ex. $2x + 3y = 6$
 - Slope-Intercept form: $y = mx + b$ (m is the slope/rate of change and b is the y intercept) Ex. $y = 2x + 1$
 - Point-Slope form: $y - y_1 = m(x - x_1)$ or $y = m(x - x_1) + y_1$ (m is the slope and (x_1, y_1) is a point on the line). Ex. $y = 2(x - 1) + 3$
- Linear word problems will include a rate of change that is constant. If you were to represent the problem with a table, you would see a common difference. If you graph the context, the graph will be a line with a slope. The situation can be represented by a linear equation.

Examples:

A **Linear** Equation has **no exponent** on a variable:

$2x + y - z = 4$ ✓ **linear**

$2x + y^2 - z = 4$ ✗ **non-linear**



x	y
2	50
4	35
6	20
8	5

+2 (between rows), -15 (between columns)

♦ **linear**

x	y
1	1
4	16
7	49
10	100

+3 (between rows), +15 (between columns)

♦ **nonlinear**

Joseph earns \$520 per week for 40 hours of work. If he works overtime, he makes \$22 per overtime hour. Find a rule for the linear function that describes his weekly salary if he works x hours of overtime. Use it to find how much Joseph earns if he works 7 hours of overtime.

\$520 for 40 hrs → PAY
 \$22 for 1 hr of overtime
 \$22.2 x

$$f(x) = \$520 + \$22x$$

$$f(x) = 520 + 22x$$

♦ **linear**

Exponential functions have several defining characteristics:

1. Their graph will be a curve. The curve will have one side that flattens out as it approaches a particular value. The other side of the curve will have a steeper bend.
2. They have a factor of change (common ratio).
3. The equation has a variable as the exponent.
 - a. $f(x) = a \cdot b^x$ (A is the y-intercept/zero term, b is the common ratio/r). Ex. $f(x) = 3 \cdot 2^x$
4. Exponential word problems often include phrases such as “doubles each time”, is “half as big each time”, or include a percent of increase or decrease. Ex. Population increases at a rate of 2.5% each year.

Examples:

The image shows handwritten notes and two tables. The notes define an exponential function as $y = a \cdot (b)^x$, where 'a' is the starting value/y-intercept and 'b' describes how the function grows (b > 1 for growth, 0 < b < 1 for decay). It also shows a graph of an exponential function $y = 3 \cdot (\frac{1}{2})^x$ and a linear function $y = x + 2$. The tables compare linear and exponential growth patterns.

X	-2	-1	0	1	2
Y	-4	0	4	8	12

Annotations for the first table: Successive outputs, constant difference (+4).

X	-2	-1	0	1	2
Y	$0.5 = \frac{1}{2}$	2	8	32	128

Annotations for the second table: constant factor (.4).

“The account was earning 4% interest per year.”
 “The number of bacteria in the test tube was doubling every 5 minutes.”
 “The amount of radioactive material is decreasing by 5.5% each year.”

The above are all examples of situations that could be modeled with exponential functions.

Video help: <http://viewpure.com/TVX9pSPJwYA?ref=bkmk>

https://learnzillion.com/lesson_plans/6663

https://learnzillion.com/lesson_plans/5773-write-an-exponential-growth-function

Go

Topic: Geometric means

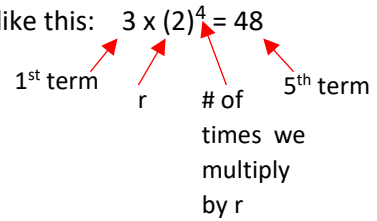
A geometric sequence has a common ratio which means that we multiply (or divide) by the same number each time.

For example, in the sequence 3, 6, 12, 24, 48, ... We multiply by 2 each time. We could keep track of this work as shown below:

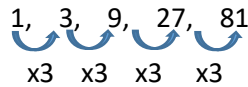
$$\begin{array}{ccccccc}
 3, & 6, & 12, & 24, & 48 & & \\
 \curvearrowright & \curvearrowright & \curvearrowright & \curvearrowright & & & \\
 \times 2 & \times 2 & \times 2 & \times 2 & & &
 \end{array}$$

We start at 3, multiply by 2 to get to 6, multiply by 2 again to get 12, multiply by 2 again to get 24, and multiply by 2 again to get 48. How many times do we multiply by 2 to get from the 1st term of 3 to the 5th term of 48? We started with 3 and multiplied by the common ratio (r) 4 times. We could write that equation like this: $3 \times (2)^4 = 48$

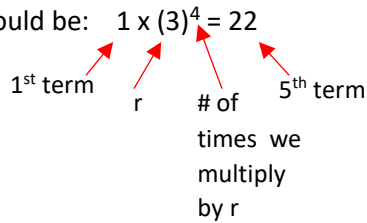
*** remember $2 \times 2 \times 2 \times 2$ can be written as 2^4



Let's try to write an equation for another geometric sequence.



Our first term, $f(1)$ is 1 and our 5th term, $f(5)$ is 81. We are multiplying by 3 each time, so our common ratio is $r=3$. We had to multiply by the common difference 4 times to get from the 1st term to the 5th term (if we take $5^{\text{th}} - 1^{\text{st}}$ we get 4). So our equation would be: $1 \times (3)^4 = 81$



If we generalize these equations we get: starting term value $\times (r)^{n - \text{starting term \#}} = \text{ending term}$

For the previous examples, we were given several terms in the sequence so we could easily find the common ratio. How can we use the equation we wrote to find r and missing terms if we don't have the entire sequence?

Let's try an example:

x (term number)	1	2	3	4
y (term value)	2			128



In this sequence we have the 1st & 4th terms, but we are missing terms 2 & 3. How can we find the common ratio (r) and the missing terms? We can use the equation: starting term value $\times (r)^{n - \text{starting term \#}} = \text{ending term}$. In this case our 1st term is 2, our 4th term is 128 and we will have to multiply by r 3 times. Our equation will look like this:

$$2 \times r \times r \times r = 128 \text{ or } 2(r)^3 = 128$$

Now we can solve this equation for r to find the common difference. Once we have r , we can use it to find the missing terms.

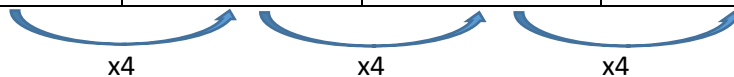
$$2(r)^3 = 128 \quad \text{Starting equation}$$

$$(r)^3 = 64 \quad \text{Divide by 2}$$

$$r = 4 \quad \text{Use mental math, a calculator, or roots to answer the}$$

We know now that the common ratio is 4, so we can find the missing terms by multiplying by 4. question "what number to the 3rd power is equal to 64?"

x (term number)	1	2	3	5
y (term value)	2	8	32	128



Let's try another example:

x (term number)	1	2	3	4	5
y (term value)	16			2	



In this sequence we have the 1st & 4th terms, but we are missing terms 2, 3, and 5. How can we find the common ratio (r) and the missing terms? We can use the equation: starting term value $\times (r)^{n - \text{starting term \#}} = \text{ending term}$. In this case our 1st term is 16, our 4th term is 2 and we will have to multiply by r 3 times. Our equation will look like this:

$16 \times r \times r \times r = 2$ or $16 \times (r)^3 = 2$ Notice, that we used the 4th term this time. We can use any 2 terms as the starting and ending term in our equation as long as we remember to count how many times we multiply by r.

Now we can solve this equation for r to find the common ratio. Once we have r, we can use it to find the missing terms.

$$16 \times (r)^3 = 2 \quad \text{Starting equation}$$

$$(r)^3 = \frac{1}{8} \quad \text{Divide by 16}$$

$$r = \frac{1}{2}$$

Use mental math, a calculator, or roots to answer the question "what number to the 3rd power is equal to $\frac{1}{8}$?" It is helpful to look at the top and bottom of the fraction separately.

We know now that the common difference is 5 so we can find the missing terms by adding 5.

x (term number)	1	2	3	4	5
y (term value)	16	8	4	2	1



In the previous two examples, we had an odd number "jumps" between terms. Does the process change if we have an even number of "jumps"? Let's take a look.

x (term number)	1	2	3	4	5
y (term value)	2				162



In this sequence we have the 1st & 5th terms, but we are missing terms 2, 3, and 4. How can we find the common ratio (r) and the missing terms? We can use the equation: starting term value $\times (r)^{n - \text{starting term \#}} = \text{ending term}$. In this case, our 1st term is 2, our 5th term is 162 and we will have to multiply by r 4 times. Our equation will look like this:

$$2 \times r \times r \times r \times r = 162 \quad \text{or} \quad 2(r)^4 = 162$$

Solve for r:

$$2(r)^4 = 162 \quad \text{Starting equation}$$

$$(r)^4 = 81 \quad \text{Divide by 2.}$$

$$r = \pm 3$$

Use mental math, calculators, or roots to answer the question "what number to the 4th power is equal to 81?"

***Notice there are 2 answers. This is because

$$3 \times 3 \times 3 \times 3 = 81 \quad \text{AND} \quad (-3) \times (-3) \times (-3) \times (-3) = 81$$

If we use $r = 3$ the missing terms would be:

x (term number)	1	2	3	4	5
y(term value)	2	6	18	54	162

$x3$ $x3$ $x3$ $x3$

If we use $r = -3$ the missing terms would be:

x (term number)	1	2	3	4	5
y(term value)	2	-6	18	-54	162

$x(-3)$ $x(-3)$ $x(-3)$ $x(-3)$

*** When r is raised to an **even** power, we will have 2 possible values for the common ratio.

Let's take a look at some even exponents to see what happens:

$$(2)^2 = 2 \times 2 = 4 \quad \text{vs.} \quad (-2)^2 = (-2) \times (-2) = 4 \quad \text{***A negative times a negative gives a positive answer}$$

$$(5)^4 = 5 \times 5 \times 5 \times 5 = 625 \quad \text{vs.} \quad (-5)^4 = (-5) \times (-5) \times (-5) \times (-5) = 625$$

A negative times a negative = positive A negative times a negative = positive

Video Examples: <https://youtu.be/YfQdcfmDNmA>

<https://youtu.be/rdFvYOZO2j8>

2.4 Ready, Set, Go!

Ready

Topic: Comparing arithmetic and geometric sequences

When you have two consecutive numbers (numbers next to each other) in an arithmetic sequence, you can find the constant difference. You will need to subtract the numbers in the order newer – older. For example, if I have the 5th and 6th terms I would find the constant difference by taking the 6th term – 5th term. I can then use the constant difference to find missing terms in the sequence.

Example: Given the arithmetic sequence: 5, 9, __, 17, 21, __, ... Find the constant difference and the missing terms.

Step 1: We have consecutive terms (1st term = 5 and 2nd term = 9) so we can subtract: $9 - 5 = 4$. Our constant difference is 4.

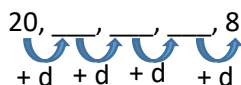
Step 2: We can use the constant difference to find the missing terms: 5, 9, 13, 17, 21, 25, ...

What if we don't have consecutive terms? We need to see what number we would add each time to get from our starting term to our ending term.

Example: Given the arithmetic sequence: 20, __, __, __, 8

Find the constant difference and the missing terms.

Step 1: We do NOT have consecutive terms because 20 is the 1st term and 8 is the 5th term. We will need to figure out what number we can add (or subtract) to get from 20 to 8. We also have to notice that we will use the constant difference 4 times (to get from the 1st term to the 5th term). We can set up an expression to help us:



We jump 4 times to get from 20 to 8

We can write this as: starting term + (# of jumps)(d) = ending term

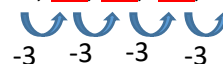
$$20 + 4(d) = 8$$

$$4d = -12 \quad \text{subtract 20 from both sides}$$

$$d = -3 \quad \text{divide by 4 on both sides}$$

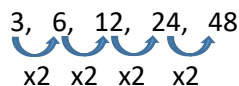
If we start with 20 and subtract 3 each time, we can get to 8.

Step 2: We can use our constant difference of -3 to fill in the missing terms: 20, 17, 14, 11, 8



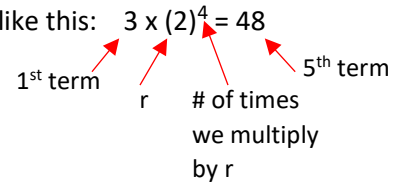
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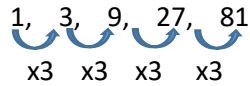


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*** remember $2 \times 2 \times 2 \times 2$ can be written as 2^4

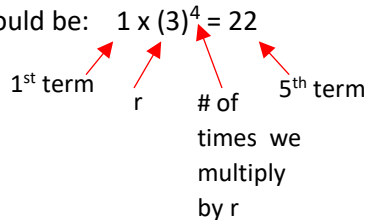


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In this sequence we have the 1st & 4th terms, but we are missing terms 2 & 3. How can we find the common ratio (r) and the missing terms? We can use the equation: starting term value $\times (r)^{n - \text{starting term \#}} = \text{ending term}$. In this case our 1st term is 2, our 4th term is 128 and we will have to multiply by r 3 times. Our equation will look like this:

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Now we can solve this equation for r to find the common difference. Once we have r , we can use it to find the missing terms.

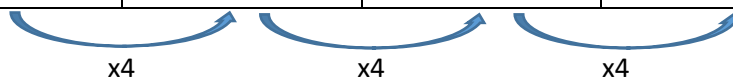
$$2(r)^3 = 128 \quad \text{Starting equation}$$

$$(r)^3 = 64 \quad \text{Divide by 2}$$

$$r = 4 \quad \text{Use mental math, a calculator, or roots to answer the question "what number to the 3rd power is equal to 64?"}$$

We know now that the common ratio is 4, so we can find the missing terms by multiplying by 4.

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Solve for r:

$$2(r)^4 = 162 \quad \text{Starting equation}$$

$$(r)^4 = 81 \quad \text{Divide by 2.}$$

$$r = \pm 3$$

Use mental math, calculators, or roots to answer the question "what number to the 4th power is equal to 81?"

***Notice there are 2 answers. This is because

$$3 \times 3 \times 3 \times 3 = 81 \quad \text{AND} \quad (-3) \times (-3) \times (-3) \times (-3) = 81$$

If we use $r = 3$ the missing terms would be:

x (term number)	1	2	3	4	5
y(term value)	2	6	18	54	162

$\xrightarrow{\quad \times 3 \quad}$
 $\xrightarrow{\quad \times 3 \quad}$
 $\xrightarrow{\quad \times 3 \quad}$
 $\xrightarrow{\quad \times 3 \quad}$

If we use $r = -3$ the missing terms would be:

x (term number)	1	2	3	4	5
y(term value)	2	-6	18	-54	162

$\xrightarrow{\quad \times (-3) \quad}$
 $\xrightarrow{\quad \times (-3) \quad}$
 $\xrightarrow{\quad \times (-3) \quad}$
 $\xrightarrow{\quad \times (-3) \quad}$

*** When r is raised to an **even** power, we will have 2 possible values for the common ratio.

Let's take a look at some even exponents to see what happens:

$(2)^2 = 2 \times 2 = 4$ vs. $(-2)^2 = (-2) \times (-2) = 4$ ***A negative times a negative gives a positive answer

$(5)^4 = 5 \times 5 \times 5 \times 5 = 625$ vs. $(-5)^4 = (-5) \times (-5) \times (-5) \times (-5) = 625$

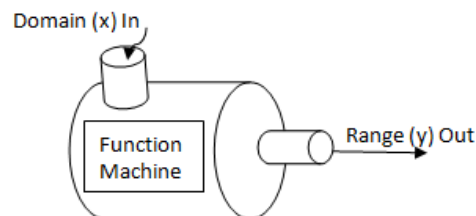
A negative times a negative = positive A negative times a negative = positive

Video Examples: https://learnzillion.com/lesson_plans/7774-compare-linear-and-geometric-growth

Set

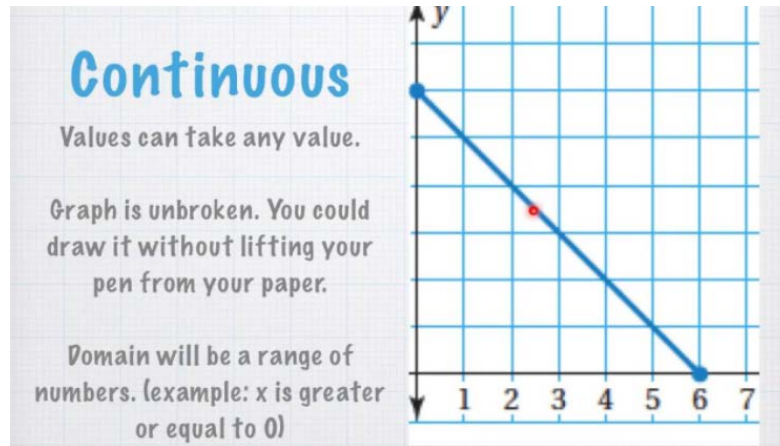
Topic: Distinguishing specifics between sequences and linear or exponential functions

We can determine whether a function is discrete or continuous by looking at the domain of the function. The domain of a function includes all of the values for x that we can use as inputs for a function. In other words, the domain is the numbers we can plug into a function. The range is the y values, or the outputs of the function. This relationship is modeled by the figure below.

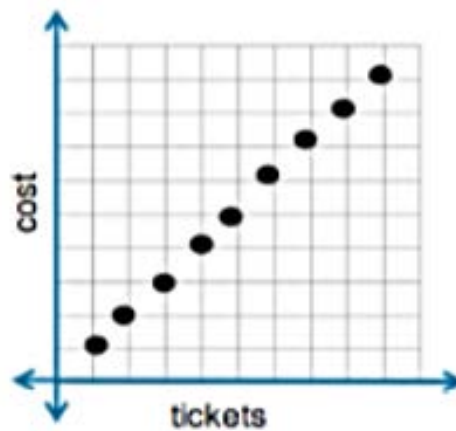


If the domain of a function includes all real numbers (or all the real numbers between the beginning and end of the function) we would consider it to be continuous. If the domain of the function only includes specific values, it would be discrete or discontinuous.

Ex. If the domain of the function is all real numbers between 0 and 6, this function would be continuous from 0 to 6.



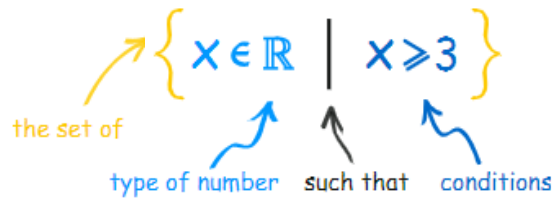
Ex. If the domain of the function can only take on certain values (like whole numbers), then the function is discrete. In the example below, the domain only includes whole numbers like 1, 2, 3, 4, 5, etc. You can't buy half of a ticket at an amusement park, so the domain can't be continuous.



We can use symbols to represent different sets of numbers. For example, instead of writing the phrase “all real numbers” each time, we can use the symbol \mathbb{R} . Here are some examples of other symbols that we can use for domain.

Symbol	In words	Description	Examples
\mathbb{R}	All real numbers	Positive, negative, zero, fractions, decimals, irrational numbers, etc.	0, 1, 2, 3, $\frac{3}{4}$, 0.125, π , $\sqrt{2}$, etc
\mathbb{W}	Whole numbers	Zero and positive numbers	0, 7, 212, and 1023
\mathbb{Z}	Integers	Negative numbers, Zero, and Positive Numbers	-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5
\mathbb{N}	Natural Numbers	“Counting” numbers not including zero	1, 2, 3, 4, 5, ...
\mathbb{Q}	Rational Numbers	Numbers you can make by dividing one integer by another (not dividing by 0)	$4 = \frac{4}{1}$; $0 = \frac{0}{1}$; $3.75 = 3\frac{75}{100}$; $\frac{1}{11} = 0.09$

We can also use special notation to write the domain of the function. This notation is called “Set Builder” notation:



We read this symbol as: “the set of all x ’s that are an element (member) of the real numbers, such that x is greater than or equal to 3.” In other words, the domain of this function is real numbers greater than or equal to 3.

Ex. $\{x \mid x \in \mathbb{N}, x < 10\}$ The domain of this function would be read “The set of all x such that x is an element of the natural numbers and x is less than 10.”

Continuous functions will include real numbers in their domain, but discrete functions will be limited to a specific set of values.

Both Geometric and Arithmetic Sequences are limited to whole number inputs (at the beginning (0 term), 1st term, 2nd term, etc.). This means that their domains are limited to a specific set of numbers, in this case, the whole numbers. For this reason, they will be discrete.

Recursive equations like $f(1) = 3$, $f(n) = f(n - 1) - 3$ [arithmetic] or $f(1) = 3$, $f(n) = f(n - 1) \cdot 3$ [geometric] represent sequences, which have a discrete domain.

Explicit equations are better representations of continuous functions. The equation $f(x) = -3x + 6$ represents a continuous linear function. The equation $f(x) = 2(3)^x$ represents a continuous exponential function. To determine if the function is linear or exponential, you can look at the format of the explicit equation:

- Linear equations in slope-intercept form: $y = mx + b$
- Exponential equations of the form: $y = a(b)^x$

Arithmetic sequences and linear functions are similar because they both have a constant rate of change. However, the domain of an arithmetic sequence is limited to whole numbers, while a linear function has a domain of all real numbers. This means that sequences will be discrete, while a linear function will be continuous.

Geometric sequences and exponential functions are similar because they both have a common ratio or factor of change. The domain of a geometric sequence is limited to whole numbers, while the domain of an exponential function is all real numbers.

Video help: <http://viewpure.com/o4G5HOvGUX4?start=0&end=0>

<http://viewpure.com/fw5ekx-C-2A?start=0&end=0>

https://learnzillion.com/lesson_plans/6127-recognize-the-characteristics-of-a-linear-function

https://learnzillion.com/lesson_plans/6663-distinguish-between-linear-and-exponential-functions-using-tables

Go

Topic: Writing explicit equations for linear and exponential models.

When you are given a table to represent a sequence and asked to write the recursive and explicit formulas, you need to start by determining which type of sequence the table represents. Once you know the type of sequence, you can write the recursive and explicit rules.

Step 1: Determine the type of sequence by determining if it has a constant difference or common ratio:

Term #: n	Term value: f(n)
1	3
2	11
3	19
4	27
5	35
...	...
n	$f(1)=3; f(n)=f(n-1) + 8$

+8
+8
+8
+8
+8

We can see from the table that we are adding 8 each time. This means that we have an arithmetic sequence with a constant difference (d) of 8.

Our recursive rule is: $f(1) = 3; f(n) = f(n - 1) + 8$

Step 2: Write the explicit function by looking at the starting term & how many groups of 8 are added each time.

Term #: n	Term value: f(n)	How many groups of 8?
1	3	3 + zero 8's
2	$11 = 3 + 8$	3 + one 8
3	$19 = 3 + 8 + 8$	3 + two 8's
4	$27 = 3 + 8 + 8 + 8$	3 + three 8's
5	$35 = 3 + 8 + 8 + 8 + 8$	3 + four 8's
...	...	
n	$3 + 8 + \dots + 8$	3 + (n - 1)8's

The repeated addition can be written as multiplication: ex. $8 + 8$ can be written as $2(8)$. We always have one group less than the term number. So for the nth term, we have $(n - 1)$ groups of 8.

Our explicit equation is:

$$F(n) = 3 + 8(n - 1)$$

For an arithmetic sequence, the explicit equation can be written as $f(n) = \text{starting term value} + d(n - \text{starting term \#})$ where d is the common difference. For the tables above, the table starts with the 1st term so the starting term # is 1, and the first term's value is 3. The common difference is 8.

Example 2: Determine whether the given information represents an arithmetic or geometric sequence. Then write the recursive and explicit equation for each.

Your brother invested \$200 into an account that earns 10% interest each year (using simple interest).

Step 1: Determine the type of sequence by determining if it has a constant difference or common ratio:

# of months n	\$ in account: f(n)
0	200
1	220
2	242
3	266.20
4	292.82
...	...
n	$f(0)=200; f(n)=f(n-1) \times 1.1$

The \$200 represents the amount "at the beginning". This means that we are starting with the 0 term. Next, we can see that the first year the account increased by \$20. The next year the account increased by \$22. Since it does not increase by the same amount, this is not an arithmetic pattern. Next, we look for a common ratio by dividing the newer output by the older output: $\frac{220}{200} = 1.1, \frac{242}{220} = 1.1, \frac{266.20}{242} = 1.1$. This means we have a geometric function with a common ratio (r) of 1.1.

Our recursive rule is: $f(0) = 200; f(n) = f(n - 1) \times 1.1$

Step 2: Write the explicit function by looking at the starting term & how many groups of 1.1 are multiplied each time.

Term #: n	Term value: f(n)	How many groups of 8?
0	200	200×1.1^0
1	$220 = 200 \times 1.1$	200×1.1^1
2	$242 = 200 \times 1.1 \times 1.1$	200×1.1^2
3	$266.20 = 200 \times 1.1 \times 1.1 \times 1.1$	200×1.1^3
4	$292.82 = 200 \times 1.1 \times 1.1 \times 1.1 \times 1.1$	200×1.1^4
...	...	
n	$200 \times 1.1 \times \dots \times 1.1$	200×1.1^n

The repeated multiplication can be written using exponents: ex. $1.1 \times 1.1 \times 1.1$ can be written as 1.1^3 . We have the same number of groups as the term number. So for the nth term, we multiply by 1.1 "n" times.

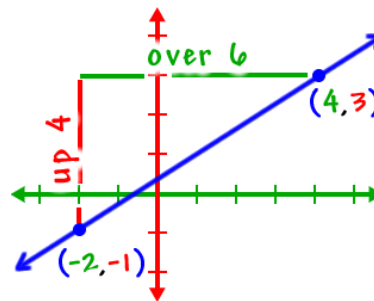
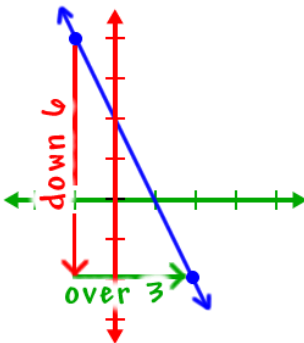
Our explicit equation is: $f(n) = 200(1.1)^n$

For a geometric sequence, the explicit equation can be written as $f(n) = \text{starting term value} \times (r)^{(n - \text{starting term \#})}$ where r is the common ratio. For the tables above, the table starts with the 0 term so the starting term # is 0, and the starting term's value is 200. The common ratio is 1.1.

When you are given a graph, you can write the slope-intercept form of a line $y = (\text{slope})(x) + y\text{-intercept}$, or as $y = mx + b$ where m represents the slope, and b represents the y-intercept.

First, let's clear up some vocabulary. The slope of the line is the ratio of the rise over run.

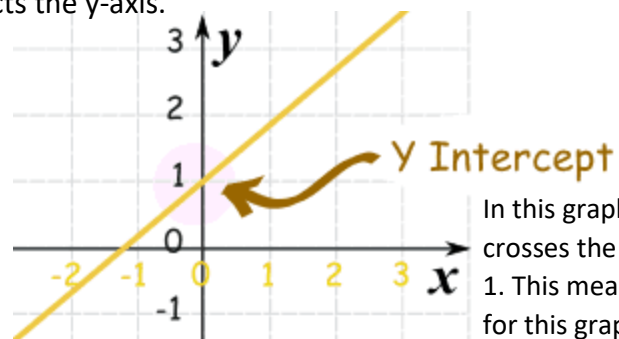
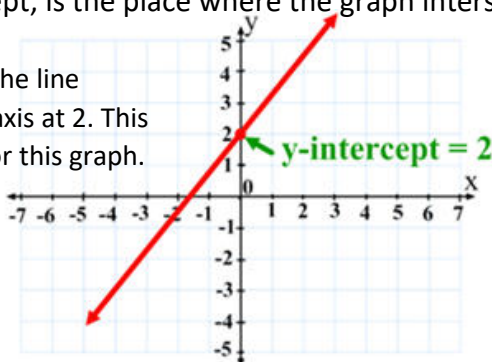
In this graph, the line moves down 6 as it goes right 3. The rise is -6 because it moves down, the run is 3 because it moves right. The slope is equal to $\frac{\text{rise}}{\text{run}} = \frac{-6}{3} = -2$, so $m = -2$



In this graph, the line moves up 4 as it goes right 6. The rise is 4 because it moves up, the run is 6 because it moves right. The slope is equal to $\frac{\text{rise}}{\text{run}} = \frac{4}{6} = \frac{2}{3}$, so $m = \frac{2}{3}$

The y-intercept, is the place where the graph intersects the y-axis.

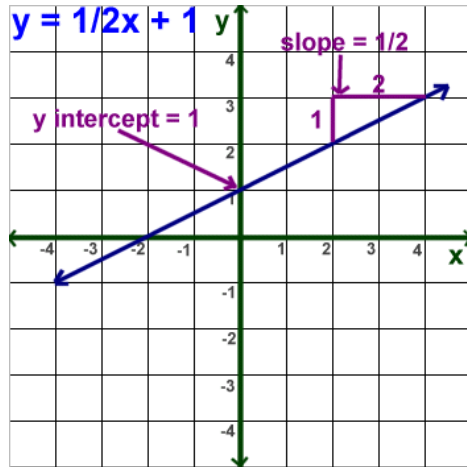
In this graph, the line crosses the y-axis at 2. This means $b = 2$ for this graph.



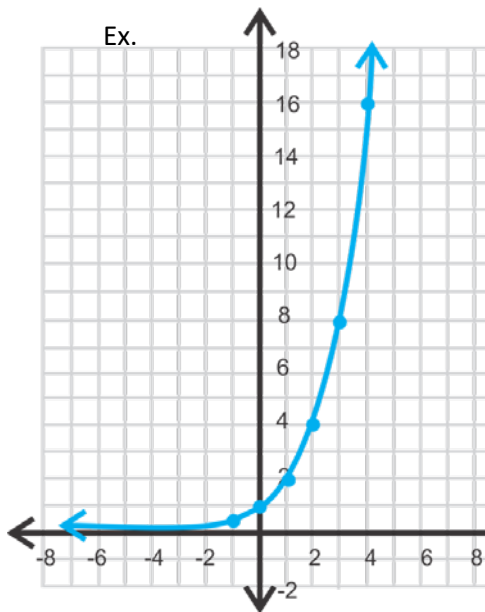
In this graph, the line crosses the y-axis at 1. This means $b = 1$ for this graph.

To write the slope-intercept equation from a graph, you will need to identify the slope (m) and the y-intercept (b) to plug into the form: $y = mx + b$.

In this graph, the line moves up 1 as it goes right 2. The rise is 1 because it moves up, the run is 2 because it moves right. The slope is equal to $\frac{\text{rise}}{\text{run}} = \frac{1}{2}$, so $m = \frac{1}{2}$. The graph crosses the y-axis at 1, so $b = 1$. If we plug them into slope-intercept form, we get: $y = \frac{1}{2}x + 1$



To write the explicit formula for an exponential function given a graph, you can use the general form of an exponential equation: $f(n) = \text{starting term value} \times (r)^{(n - \text{starting term \#})}$. It is helpful to identify a few points on the graph in order to make a table. From the table you can find a starting term number, starting term value, and common ratio.



Term #: x	Term value: $f(x)$	Exponential Rule?
0	1	1×2^0
1	$2 = 1 \times 2$	1×2^1
2	$4 = 1 \times 2 \times 2$	1×2^2
3	$8 = 1 \times 2 \times 2 \times 2$	1×2^3
4	$16 = 1 \times 2 \times 2 \times 2 \times 2$	1×2^4
...	...	
n	$1 \times 2 \times 2 \times \dots \times 2 \times 2$	1×2^n

The repeated multiplication can be written using exponents: ex. $2 \times 2 \times 2$ can be written as 2^3 . We have the same number of groups as the term number. So for the n th term, we multiply by 2 “ n ” times.

Our explicit equation is: $f(n) = \text{starting term value} \times (\text{common ratio})^{n - \text{starting term \#}}$

Our starting term # is 0, our starting term value is 1, and the common ratio is 2. Plugging these into the formula, we will get: $f(n) = 1 \times (2)^{n - 0}$ which simplifies to $f(n) = 2^n$. If we have the y-intercept, the exponent will be “ x ” or “ n ”.

Video help: https://learnzillion.com/lesson_plans/5940

https://learnzillion.com/lesson_plans/6323

https://learnzillion.com/lesson_plans/5701-writing-an-explicit-formula-for-the-graph-of-an-exponential-relationship

<http://viewpure.com/ej-zSIYKvxA?start=0&end=0>

2.5 Ready, Set, Go!

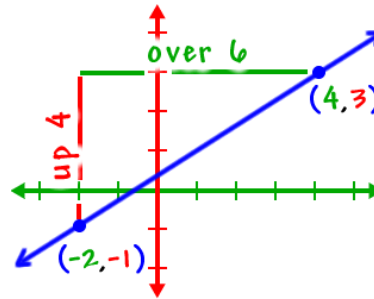
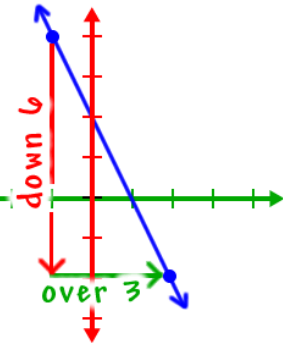
Ready

Topic: Writing equations of lines

When you are given a graph, you can write the slope-intercept form of a line $y = (\text{slope})(x) + y\text{-intercept}$, or as $y = mx + b$ where m represents the slope, and b represents the y -intercept.

First, let's clear up some vocabulary. The slope of the line is the ratio of the rise over run.

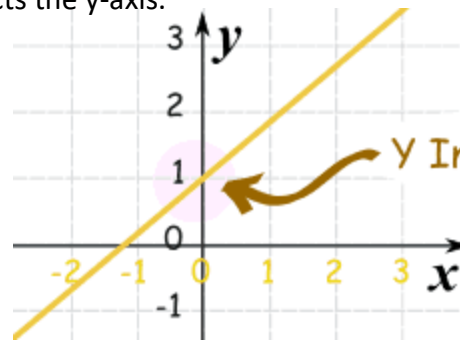
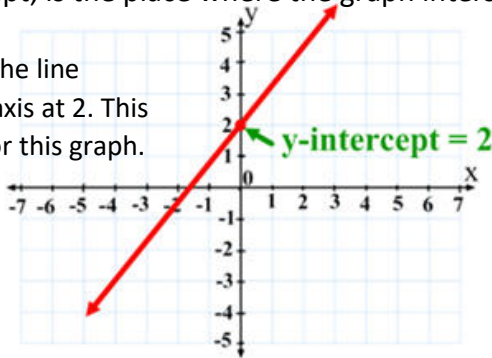
In this graph, the line moves down 6 as it goes right 3. The rise is -6 because it moves down, the run is 3 because it moves right. The slope is equal to $\frac{\text{rise}}{\text{run}} = \frac{-6}{3} = -2$, so $m = -2$



In this graph, the line moves up 4 as it goes right 6. The rise is 4 because it moves up, the run is 6 because it moves right. The slope is equal to $\frac{\text{rise}}{\text{run}} = \frac{4}{6} = \frac{2}{3}$, so $m = \frac{2}{3}$

The y -intercept, is the place where the graph intersects the y -axis.

In this graph, the line crosses the y -axis at 2. This means $b = 2$ for this graph.



In this graph, the line crosses the y -axis at 1. This means $b = 1$ for this graph.

If you are given m (slope) and b (y -intercept), you can substitute them into the equation $y = mx + b$.

Ex. Write the slope-intercept form of the line ($y = mx + b$) if $m = -2$ and $b = 3$

Equation:

$$y = -2x + 3$$

Substitute -2
in for m

Substitute 3 in for b .
It's addition because
3 is positive.

Ex. Write the slope-intercept form of the line ($y = mx + b$) if $m = -\frac{1}{2}$ and $b = -5$

Equation:

$$y = -\frac{1}{2}x - 5$$

Substitute $-\frac{1}{2}$
in for m

Substitute -5 in for b . We
use subtraction because
the 5 is negative.

Ex. Write the slope-intercept form of the line ($y = mx + b$) if $m = 4$ and $b = \frac{3}{7}$

Equation: $y = 4x + \frac{3}{7}$

Substitute $-\frac{1}{2}$ in for m

Substitute -5 in for b . We use subtraction because the 5 is negative.

Another form of a linear function that uses the slope is point-slope form. Most textbooks will show point-slope form in the following format:

Output	Slope	Input
↓	↓	↓
$y - y_1 = m(x - x_1)$		
	↑	↑
y-coordinate of a point on the line		x-coordinate of the same point on the line

In Math 1, we will modify the point-slope form slightly by adding y_1 to the right-side of the equation like this:

$y = m(x - x_1) + y_1$ This allows us to have the ordered pair (x_1, y_1) together on one side of the equation and also matches the transformation form of other types of functions. In both of these equations, we will need the slope (m) and an ordered pair/point (x_1, y_1) . This form of the equation is useful when you know the location of a point on the line other than the y -intercept

POINT-SLOPE FORM

$y - y_1 = m(x - x_1)$
 given $m, (x_1, y_1)$

$m = -1$ $(3, -2)$

$y - -2 = -1(x - 3)$

$y + 2 = -(x - 3)$

$y = -(x - 3) - 2$

****Notice that the x -coordinate is **positive**
3. The general form has subtraction inside the parentheses, so when you substitute the 3 into the equation, you will have $(x - 3)$.

Ex. Write the point-slope form of the linear function with a slope of $m = \frac{1}{2}$ and the point $(-2, -4)$.

Begin with the general point-slope form: $y - y_1 = m(x - x_1)$

Substitute $\frac{1}{2}$ in for m , -2 for x_1 , and -4 for y_1 : $y - -4 = \frac{1}{2}(x - -2)$

Simplify by cleaning up the signs: $y + 4 = \frac{1}{2}(x + 2)$

Move the 4 to the right-side by subtracting:

$y = \frac{1}{2}(x + 2) - 4$

Ex. Write the point-slope form of the linear function with a slope of $m = -3$ and the point $(0, 5)$.

Begin with the general point-slope form: $y - y_1 = m(x - x_1)$

Substitute -3 in for m , 0 for x_1 , and 5 for y_1 : $y - 5 = -3(x - 0)$

Move the 5 to the right-side by adding: $y = -3(x - 0) + 5$

Sometimes we are asked to write the point-slope form of the equation from two points. In this case, we will need to find the slope of the line before we can write the equation. There are two methods you can use to find the slope. The first

method is to use the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ and substitute your coordinate points into the formula. The second

method is to use the two points to create a slope triangle to find the $\frac{\text{rise}}{\text{run}}$.

Method 1: Find the slope of the line through the points $(2, 1)$ and $(6, 3)$. The first coordinate listed is your (x_1, y_1) and the second coordinate listed is (x_2, y_2) . So in this example we have the following:

$x_1 = 2$, $y_1 = 1$, $x_2 = 6$, and $y_2 = 3$. We will substitute these values into our slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Begin with the general slope form: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Substitute $x_1 = 2$, $y_1 = 1$, $x_2 = 6$, and $y_2 = 3$: $m = \frac{3 - 1}{6 - 2}$

Simplify: $m = \frac{2}{4} = \frac{1}{2}$

****Notice that BOTH y coordinates are on top and BOTH x coordinates are in the denominator.

Now that we have our slope, we can choose either point and write the point-slope equation:

Begin with the general point-slope form: $y - y_1 = m(x - x_1)$

Substitute $\frac{1}{2}$ in for m , 2 for x_1 , and 1 for y_1 : $y - 1 = \frac{1}{2}(x - 2)$

Move the 1 to the right-side by adding: $y = \frac{1}{2}(x - 2) + 1$

Or, if we choose the point $(6, 3)$:

Begin with the general point-slope form: $y - y_1 = m(x - x_1)$

Substitute $\frac{1}{2}$ in for m , 6 for x_1 , and 3 for y_1 : $y - 3 = \frac{1}{2}(x - 6)$

Move the 3 to the right-side by adding: $y = \frac{1}{2}(x - 6) + 3$

***Both equations are correct.

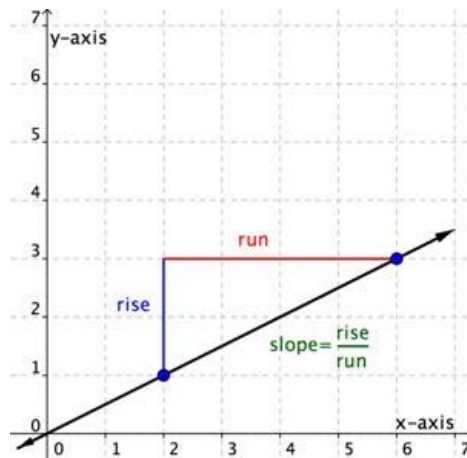
Method 2: Find the slope of the line through the points (2, 1) and (6, 3) using a slope triangle.

Sketch a quick graph of the points to create a slope triangle (as shown in the graph to the right):

The slope will be the $\frac{\text{rise}}{\text{run}}$

For the points (2, 1) and (6, 3), we will rise up 2 (positive 2) and run right 4 (positive 4).

So the slope will be $m = \frac{2}{4} = \frac{1}{2}$



Once we have the slope, we will write the point slope equation using either of the given points:

$$y - y_1 = m(x - x_1)$$

or

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{2}(x - 2)$$

$$y - 3 = \frac{1}{2}(x - 6)$$

$$y = \frac{1}{2}(x - 2) + 1$$

$$y = \frac{1}{2}(x - 6) + 3$$

***Both equations are correct.

Video Examples: <http://viewpure.com/fr9w7PCxDGE?start=0&end=0>

<http://virtualnerd.com/algebra-1/linear-equation-analysis/point-slope-standard-form/point-slope-examples/point-slope-form-definition>

Note: for these two videos, the equation is left with the y_1 term on the left. For our curriculum, you will want to do one extra step to move the y_1 term to the right-hand side of the equation.

<http://viewpure.com/zihsQC0IUd8?start=0&end=0>

<http://viewpure.com/iVvGbDr-7lQ?start=0&end=0>

Set

Topic: Graphing linear and exponential functions

We can first check if a situation is linear (arithmetic) by looking for a constant difference. If the situation involves increasing or decreasing by the same amount each time, it is linear. If the context requires multiplying (or dividing) by a constant factor each time, it is exponential (geometric).

Ex. Each week you earn \$5 by doing chores around the house. In this case, you are adding \$5 each week. The rate of change is constant so the context is linear (arithmetic).

Ex. Each month you earn 3% interest on your savings. We calculate percent by multiplying, so this is an exponential (geometric) context.

***In a context, you can look for key terms that will help you identify if the situation can be modeled by a linear or exponential equation.

Linear: Miles per hour, feet per second, price for each item, increase by 5 each time, 3 units smaller, etc.

Exponential: Increase by a percent (%), doubles, triples, half as much, etc.

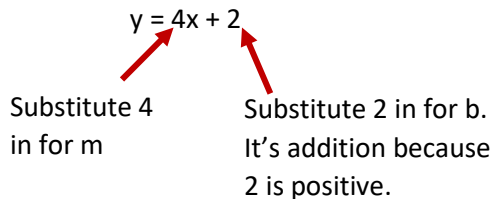
When you are given a linear context, you can write the slope-intercept form of a line $y = (\text{slope})(x) + y\text{-intercept}$, or as $y = mx + b$ where m represents the slope, and b represents the y -intercept.

Example: Starting value is 2 and the function is 4 units larger each time.

In this case, we are increasing by 4 each time, so we have a constant rate of change (slope). The phrase “starting value” indicates that when x is 0, y is 2. This represents our y -intercept of 2. The phrase “4 units larger each time” implies that we are moving up 1 whole unit at a time. This means that our graph will be discrete (we’ll only use whole number inputs).

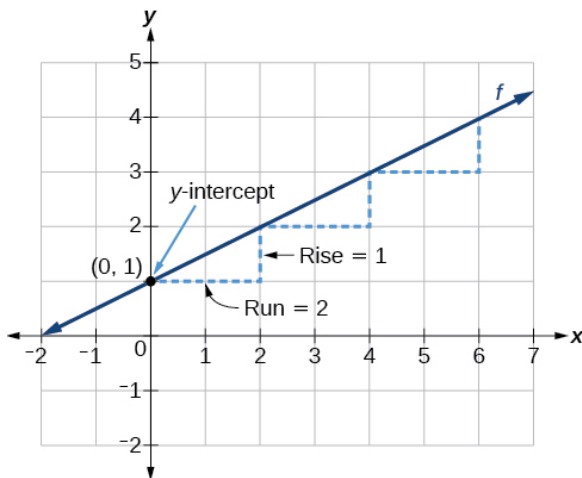
If you are given m (slope) and b (y -intercept), you can substitute them into the equation $y = mx + b$.

Equation: Our slope (m) is 4, and our y -intercept (b) is 2.

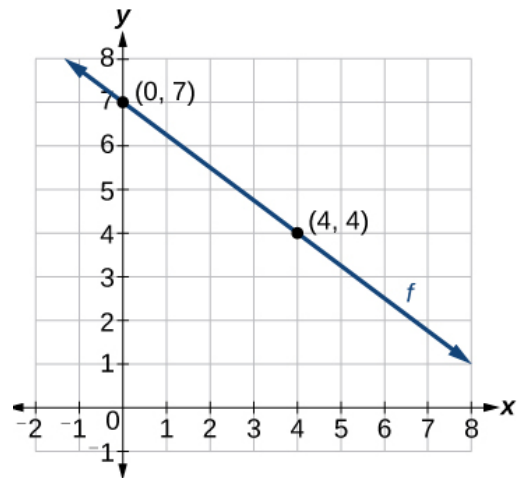


Once we have the equation, we can graph using the y -intercept as a starting point, and counting by the slope to find another point on the graph. We can continue plotting points using the slope. To determine how to label our axes, we can look at the slope.

For a positive slope, we will count up and right, so we can place our axis further to the left as shown:



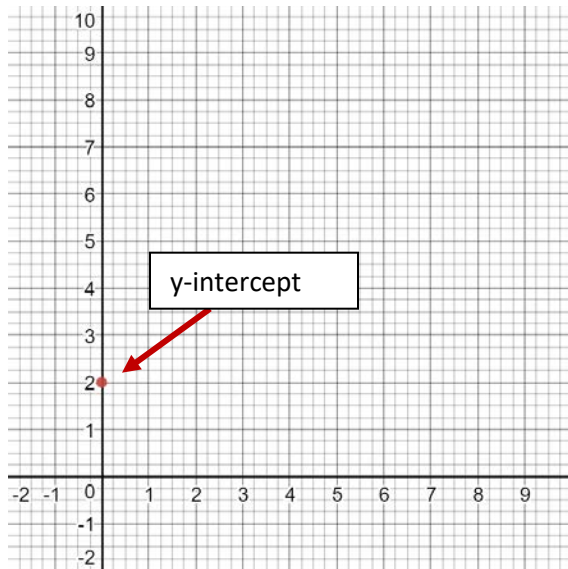
For a negative slope, we will count down and right, so we can place our axis further to the left as shown:



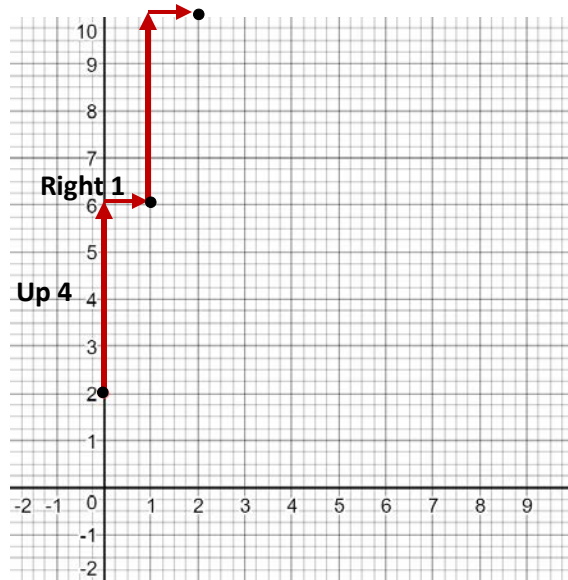
*** We want to place the x and y axis in a location that allows us to graph the y -intercept and at least one additional point on the line.

For the example above, we were able to write the equation $y = 4x + 2$ to represent the scenario “the starting value is 2 the function is 4 units larger each time.” To graph this equation we will need to first plot the y-intercept, and then use the slope to find another point.

Step 1: Plot the y-intercept (0, 2)



Step 2: Start at the y-intercept and use the slope 4 to count up 4, right 1. Repeat to find more points.



For an exponential situation, you can use the general form of an exponential equation:

$f(x) = a \cdot r^x$ where a is the starting term (y-intercept) and r is the common ratio.

Example: Starting value is 16 and the function is half as big each time.

In this case, we are taking $\frac{1}{2}$ of the previous term each time, so we have a common ratio (r). The phrase

“starting value” indicates that when x is 0, y is 16. This represents our y-intercept of 16. The phrase “half as big each time” implies that we using whole number inputs. This means that our graph will be discrete.

If you are given the common ratio (r) and the y-intercept, you can substitute them into the equation:

$$f(x) = a \cdot r^x .$$

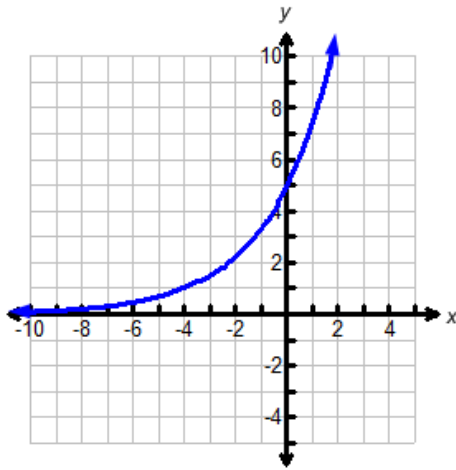
Equation: Our common ratio (r) is $\frac{1}{2}$, and our starting amount (a) is 16.

$$f(x) = 16 \left(\frac{1}{2} \right)^x$$

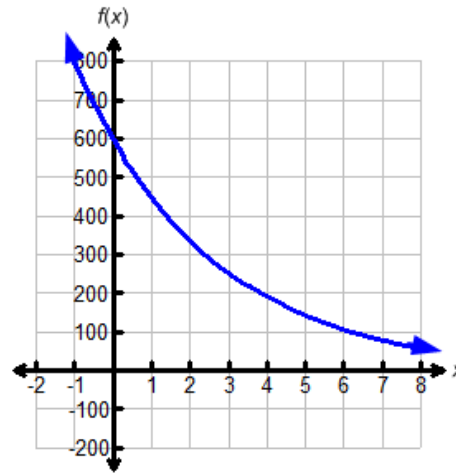
Substitute 16
in for a
Substitute $\frac{1}{2}$ in for r .

Once we have the equation, we can graph using the y-intercept as a starting point, and using the common ratio to find the next term. We can continue plotting points using the common ratio. To determine how to label our axes, we can look at the starting term and common ratio.

When the common ratio is greater than 1, the graph will move up and right, so we can place our axis as shown:



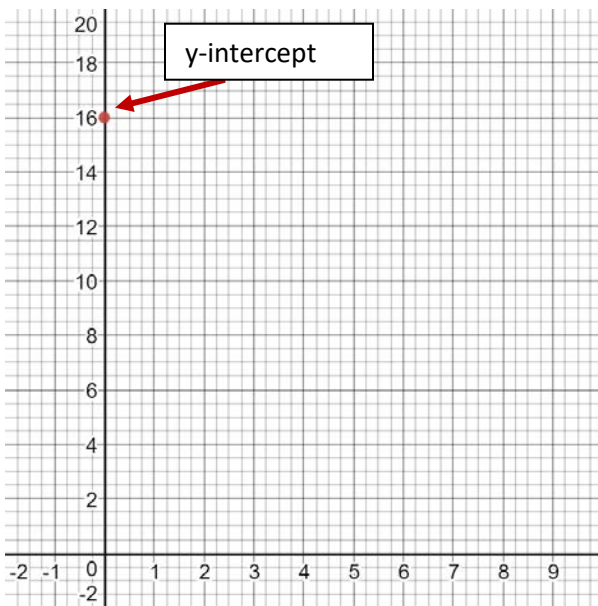
When the common ratio is between 0 and 1, the curve will move down and right, so we can place our axis as shown:



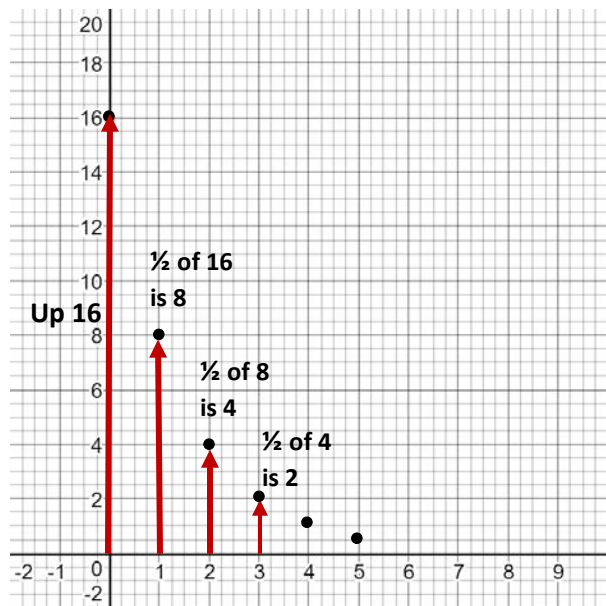
*** We want to place the x and y axis in a location that allows us to graph the y-intercept and at least three additional points so we can see the curve. Exponential functions grow and decay quickly, so we may need to scale our graph to fit enough data points to see the curve.

For the example above, we were able to write the equation $f(x) = 16\left(\frac{1}{2}\right)^x$ to represent the scenario “starting value is 16 and the function is half as big each time.” To graph this equation we will need to first plot the y-intercept, and then use the common ratio to find another point.

Step 1: Plot the y-intercept (0, 16)



Step 2: Start at the y-intercept and move up half the height of the previous point as you move to the right 1.



Video help: <http://viewpure.com/vGNSMUKEQ9c?start=0&end=0>

<http://viewpure.com/GGytywqpGXA?start=0&end=0>

Go

Topic: Equivalent equations

To prove that both sides of an equation are equivalent, we can first define some common vocabulary:

Variable: A variable is a symbol for an unknown value.

Coefficient: A number that is multiplied by a variable.

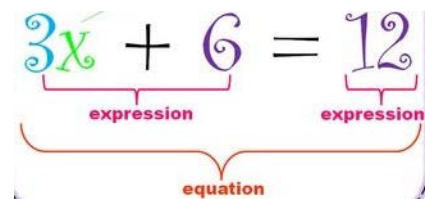
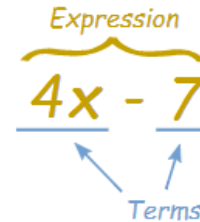
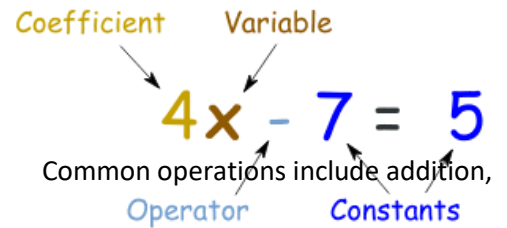
Operator: A symbol that shows us what operations are being done. subtraction, multiplication, division, etc.

Constant: A number that is not multiplied by a variable.

Term: A number, variable, or product of a number and variable.

Expression: A combination of a single term or terms using operations like addition and subtraction.

Equation: A statement that two expressions are equal.



Some common steps to solving equations:

Like Terms: Terms that have the same variables to the same powers. Like terms can be combined through addition or subtraction.

SAME VARIABLES to the SAME POWER

$$3a \quad 12a \quad 54a \quad 5x \quad 12x \quad 0.9x \\ 3xy^2 \quad 9xy^2 \quad 12xy^2$$

Commutative Property: The order in which you **add** or **multiply** does not matter. (This property does NOT apply to subtraction or division) This allows you to rearrange expressions to change the order.

Commutative Property	
Of Addition	Of Multiplication
$a + b = b + a$	$a \cdot b = b \cdot a$
Examples	Examples
$2 + 3 = 3 + 2$	$2 \cdot 3 = 3 \cdot 2$
$(2+3)^3 + 5 = 5 + (2+3)^3$	$(2+3)^3 \cdot 5 = 5 \cdot (2+3)^3$
$3x^2y + 2xyz = 2xyz + 3x^2y$	$3x^2y \cdot 2xyz = 2xyz \cdot 3x^2y$

Distributive Property: Multiplication distributes over addition (or subtraction). This means that you can multiply a number by a sum (or difference) of 2 terms by distributing the multiplier outside the parentheses to each addend inside the parentheses. (This property is useful when we have a variable added to a constant inside the parentheses. In this case, we wouldn't be able to add inside the parentheses first.)

$$a(b + c) = ab + ac \\ 2(3 + 4) = 2(3) + 2(4)$$

To determine if 2 expressions are equivalent, we can simplify both expressions and compare them. We will use distributive property, commutative property, and will combine like terms to get both expressions in the same form.

When you are asked to prove that 2 expressions are equivalent, you will need to make both sides match.

Example: $-8x + 1 = 3x + 4 - 3(2x + 1) - 5x$

In this example, we want to simplify the right-hand side to make it match the left side.

$$-8x + 1 = 3x + 4 - 3(2x + 1) - 5x$$

We can start with distributive property. Notice that the subtraction sign means that we will multiply by a negative 3.

$$-8x + 1 = 3x + 4 - 6x - 3 - 5x$$

Identify the like terms (this includes the sign in front of each term). Combine them by adding or subtracting.

$$-8x + 1 = -8x + 1$$

When we are done simplifying, we can see that the right-hand side matches the left-hand side. This means that the expressions are equivalent.

Video help: <http://viewpure.com/gw893STHN9w?start=0&end=0>

<http://viewpure.com/T0a92gEDukY?start=0&end=0>

<https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-equivalent-exp/cc-6th-combining-like-terms/e/combining-like-terms-0.5> (combining like terms practice)

<https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-equivalent-exp/cc-6th-combining-like-terms/e/combining-like-terms-with-distribution> (distributing and combining like terms practice)

***Note that these videos will just demonstrate what to do on the right-hand side of the equation. You will need to simplify first and then compare the two sides.

2.6 Ready, Set, Go!

Ready

Topic: Comparing linear and exponential models

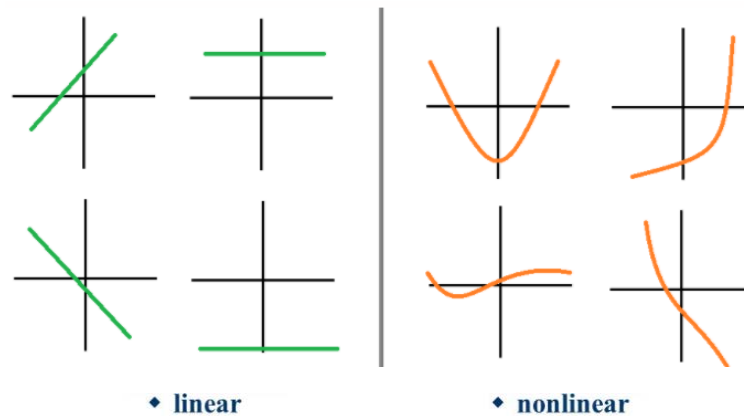
Linear functions have several defining characteristics:

1. Their graph will be a straight line (or segment depending on the context).
2. They have a constant rate of change (slope/common difference).
3. The equation is a first degree equation (x is to the 1st power so we typically don't write the exponent).
 - a. Standard form: $Ax + By = C$ (A, B, and C are whole numbers and A is positive). Ex. $2x + 3y = 6$
 - b. Slope-Intercept form: $y = mx + b$ (m is the slope/rate of change and b is the y intercept) Ex. $y = 2x + 1$
 - c. Point-Slope form: $y - y_1 = m(x - x_1)$ or $y = m(x - x_1) + y_1$ (m is the slope and (x_1, y_1) is a point on the line). Ex. $y = 2(x - 1) + 3$
4. Linear word problems will include a rate of change that is constant. If you were to represent the problem with a table, you would see a common difference. If you graph the context, the graph will be a line with a slope. The situation can be represented by a linear equation.

Examples:

A **Linear** Equation has **no exponent** on a variable:

$2x + y - z = 4$ ✓ **linear**
 $2x + y^2 - z = 4$ ✗ **nonlinear**



x	y
2	50
4	35
6	20
8	5

+2, -15, +2, -15, +2, -15

◆ **linear**

x	y
1	1
4	16
7	49
10	100

+3, +15, +3, +33, +3, +51

◆ **nonlinear**

Joseph earns \$520 per week for 40 hours of work. If he works overtime, he makes \$22 per overtime hour. Find a rule for the linear function that describes his weekly salary if he works x hours of overtime. Use it to find how much Joseph earns if he works 7 hours of overtime.

\$520 for 40 hrs → PAY
 \$22 for 1 hr of overtime
 \$22 · x x

$$f(x) = \$520 + \$22x$$

$$f(x) = 520 + 22x$$

◆ **linear**

Exponential functions have several defining characteristics:

1. Their graph will be a curve. The curve will have one side that flattens out as it approaches a particular value. The other side of the curve will have a steeper bend.
2. They have a factor of change (common ratio).
3. The equation has a variable as the exponent.
 - a. $f(x) = a \cdot b^x$ (A is the y-intercept/zero term, b is the common ratio/r). Ex. $f(x) = 3 \cdot 2^x$
4. Exponential word problems often include phrases such as “doubles each time”, is “half as big each time”, or include a percent of increase or decrease. Ex. Population increases at a rate of 2.5% each year.

Examples:

My Definition: An exponential function is a function that has a variable in the exponent.

Vocabulary word: Exponential Functions

Example: $y = 3 \cdot (\frac{1}{2})^x$

Non-Example: $y = x + 2$ (linear)

Characteristics: $0 < b < 1$ decay, $b > 1$ growth. $y = a \cdot (b)^x$ describes how the function grows. Starting value y-intercept.

X	-2	-1	0	1	2
Y	-4	0	4	8	12

Successive outputs, constant difference (+4)

X	-2	-1	0	1	2
Y	$0.5 = \frac{1}{2}$	2	8	32	128

constant factor (.4)

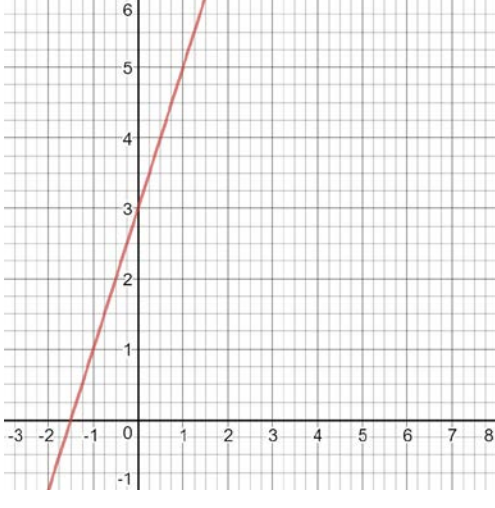
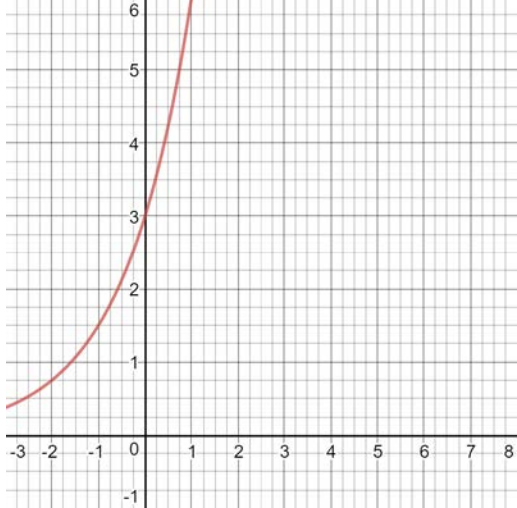
- "The account was earning 4% interest per year."
- "The number of bacteria in the test tube was doubling every 5 minutes."
- "The amount of radioactive material is decreasing by 5.5% each year."

The above are all examples of situations that could be modeled with exponential functions.

We can compare linear and exponential functions by looking at several key features:

Type of growth, the type of sequence that corresponds to the function, table, rate/factor of change, graph, and the y-intercept.

	$y = 3 + 2x$	$y = 3(2^x)$																								
Type of growth	Linear because there is no exponent on the variable.	Exponential because the variable is the exponent.																								
Type of sequence	This function adds 2 each time, so it is an arithmetic sequence. The constant difference is 2.	This function multiplies by 2 each time, so it is a geometric sequence. The common ratio is 2.																								
Table	<table border="1"> <thead> <tr> <th>X</th> <th>Y = 3 + 2x</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>3 + 2(0) = 3</td> </tr> <tr> <td>1</td> <td>3 + 2(1) = 5</td> </tr> <tr> <td>2</td> <td>3 + 2(2) = 7</td> </tr> <tr> <td>3</td> <td>3 + 2(3) = 9</td> </tr> <tr> <td>4</td> <td>3 + 2(4) = 11</td> </tr> </tbody> </table>	X	Y = 3 + 2x	0	3 + 2(0) = 3	1	3 + 2(1) = 5	2	3 + 2(2) = 7	3	3 + 2(3) = 9	4	3 + 2(4) = 11	<table border="1"> <thead> <tr> <th>X</th> <th>y = 3(2^x)</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>3 x 2^0 = 3</td> </tr> <tr> <td>1</td> <td>3 x 2^1 = 6</td> </tr> <tr> <td>2</td> <td>3 x 2^2 = 12</td> </tr> <tr> <td>3</td> <td>3 x 2^3 = 24</td> </tr> <tr> <td>4</td> <td>3 x 2^4 = 48</td> </tr> </tbody> </table>	X	y = 3(2^x)	0	3 x 2^0 = 3	1	3 x 2^1 = 6	2	3 x 2^2 = 12	3	3 x 2^3 = 24	4	3 x 2^4 = 48
X	Y = 3 + 2x																									
0	3 + 2(0) = 3																									
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3	3 x 2^3 = 24																									
4	3 x 2^4 = 48																									
Rate/Factor of Change	Rate of change (slope) = +2	Factor of change (common ratio) = x2 (times 2)																								

Graph		
Find the y-intercept	The y-intercept is (0, 3). This is the starting value for the equation: $y = 3 + 2x$. If you plug 0 in for x and solve, you will get the y-intercept.	The y-intercept is (0, 3). This is the starting value for the equation: $y = 3(2^x)$. If you plug 0 in for x and solve, you will get the y-intercept.

Video Examples: <http://viewpure.com/tZbyagXKKc8?start=0&end=0> (up to 9 minutes, 14 seconds)

<http://viewpure.com/Qi9fzQBpWQc?start=0&end=0>

https://learnzillion.com/lesson_plans/6663-distinguish-between-linear-and-exponential-functions-using-tables

https://learnzillion.com/lesson_plans/7695

<http://viewpure.com/vGNsmUKEQ9c?start=0&end=0>

<http://viewpure.com/xLhfSBoPtGk?start=0&end=0> (just focus on linear y-intercepts)

<http://viewpure.com/UyyUgTElJAE?start=0&end=0>

Set

Topic: Efficiency with different forms of linear and exponential functions

When we are given a context and asked to write an equation, we first need to determine if the situation can be modeled by linear growth or exponential growth. If the situation increases or decreases by a constant amount, the situation is linear. If the situation increases or decreases by a percent of the previous amount, or uses phrases like “doubles” or “half as much”, the situation is exponential.



Ex. You have \$30 and each time you complete a chore, you earn \$2. In this case, you start at 30 and add \$2 for each chore you complete. This would be an example of linear growth.

Ex. You have \$30 and each month, you earn 2% interest. To find out how much money we have each month, we would need to multiply the previous month by 1.02 (1 represents 100% of the previous amount plus the 2% interest, so we have 102% increase or 1.02). This means that the situation is an example of exponential growth.

For situations that have linear growth, we need to determine which form of the line best models the situation.

	Point-Slope Form	Slope-Intercept Form
General Equation:	$y = m(x - x_1) + y_1$	$y = mx + b$
Information Needed:	You will need the slope (m) and a point (x_1, y_1) .	You will need the slope (m) and the y-intercept (b). The y-intercept will be in the form $(0, b)$
Equation Examples:	<p>Given $m = 2$, and point $(3, 4)$:</p> $y = 2(x - 3) + 4$ <p>Given $m = -\frac{1}{2}$, and point $(-4, -6)$:</p> $y = -\frac{1}{2}(x + 4) - 6$	<p>Given $m = 2$, and the y-intercept $(0, 4)$:</p> $y = 2x + 4$ <p>Given $m = -\frac{1}{2}$, and the y-intercept $(0, -6)$:</p> $y = -\frac{1}{2}x - 6$
Context Examples:	<p>Kareem is cataloging books for the library at a rate of 25 books per day. At end of the 12th day, there are 475 books left to be cataloged. How can I write an equation to model how many books are left?</p> <ol style="list-style-type: none"> Determine which form of the linear function to use by looking at the given information. For this example, we will use point-slope form. Identify the slope. In this case, he's cataloging 25 books per day, but the number of books left is getting <u>smaller</u> each day. So $m = -25$ Identify an input and output. The days are your input and the number of books left are the outputs. So we have the point $(12, 475)$. Substitute the values into the general form: $y = m(x - x_1) + y_1$ Simplify if necessary: $y = -25(x - 12) + 475$ 	<p>Kareem is cataloging books for the library at a rate of 25 books per day. Kareem starts with 775 books to be cataloged. How can I write an equation to model how many books are left?</p> <ol style="list-style-type: none"> Determine which form of the linear function to use by looking at the given information. For this example, we will use slope-intercept form. Identify the slope. In this case, he's cataloging 25 books per day, but the number of books left is getting smaller each day. So $m = -25$ Identify an input and output. The days are your input and the number of books left are the outputs. The phrase "at the beginning" implies that 0 days have passed, so we have the point $(0, 775)$. This is our y-intercept. Substitute values into the general form: $y = mx + b$ Simplify if necessary: $y = -25x + 775$
When to Use:	If you are given the slope and a point. The slope will be the rate at which things change. It will be a fixed amount that the situation increases or decreases by. You will also have a relationship at a point that is NOT the starting time/amount. In the problem above, we had the number of books left on day 12 instead of day 0.	If you are given the slope the y-intercept (starting term). Clues that you have the y-intercept are the phrases "started with", "at the beginning", "currently", "initial amount", or "starting in year" The slope will be the rate at which things change. It will be a fixed amount that the situation increases or decreases by.

For situations that have exponential growth, we need to determine which form of the line best models the situation.

	Recursive Function	Explicit Function
General Equation:	$f(a) = \text{---}; f(n) = f(n-1) \cdot r$	$f(x) = a(b)^x$
Information Needed:	You will need a starting term : $f(a)$ and the common ratio , r .	You will need the initial value , a , and the factor of change , b . The initial value , a , is the y-intercept .
Equation Examples:	<p>Given $r = 2$ “doubles each time”, and starting value: $f(0) = 4$:</p> $f(0) = 4; f(n) = f(n-1) \cdot 2$ <p>Given $r = -\frac{1}{2}$ “half as much”, and starting value: $f(0) = 28$:</p> $f(0) = 28; f(n) = \frac{1}{2} f(n-1)$ <p>**note, you can write $f(n-1)$ times r, or r times $f(n-1)$ because multiplication is commutative (it works in any order).</p>	<p>Given $b = 2$ “doubles each time”, and an initial value of 4:</p> $f(x) = 4(2)^x$ <p>Given $b = -\frac{1}{2}$ “half as much”, and an initial value of 28:</p> $f(x) = 28\left(\frac{1}{2}\right)^x$ <p>**note, you cannot multiply the initial value by the factor of change due to the order of operations. You would need to raise b to the exponent first, and then multiply by the initial value.</p>
When to Use:	<p>If you have <u>discrete</u> data. For example, if you are only considering whole number inputs like the number of weeks. Discrete changes happen all at once at specific intervals.</p> <p style="text-align: center;">Large discrete changes</p>  <p>You can also use the recursive function to predict the next term. For example, if I have the first 5 terms, I could use the recursive rule to predict the 6th term.</p>	<p>If you are given continuous data. For example, filling a pool over time where the water is continuously being added to the pool. Equivalent change is ongoing.</p> <p style="text-align: center;">Equivalent continuous change</p>  <p>You can also use the explicit function to find a specific output without having to list all the previous terms. If I know the first 5 terms and I want to know the 20th term, I can use the explicit rule.</p>

Video help: <http://viewpure.com/tZbyagXKKc8?start=0&end=0> (start at 9 minutes, 14 seconds)

<http://viewpure.com/FLjsC20j-wo?start=0&end=0> ***Note for point-slope videos: In Math 1, we will modify the point-slope form slightly by adding y_1 to the right-side of the equation like this: $y = m(x - x_1) + y_1$. These videos will not show that final step.

<http://viewpure.com/JNmwbvN3Eq0?start=0&end=0> (watch up to 4 minutes, 45 seconds)

<http://viewpure.com/mcNGhrvp52s?start=0&end=0> (start at 4 minutes, 10 seconds)

<https://www.khanacademy.org/math/algebra/introduction-to-exponential-functions/graphs-of-exponential-growth/v/graphs-of-exponential-growth>

Go

Topic: Solving one-step equations with justifications

When we solve one-step equations, we can justify our work using Properties of Equality. The 4 basic operations each have their own property:

1. Multiplicative Property of Equality: we can multiply both sides of an equation by the same number.
2. Division Property of Equality: we can divide both sides of an equation by the same number.
3. Addition Property of Equality: we can add the same number to both sides of an equation.
4. Subtraction Property of Equality: we can subtract the same number from both sides of an equation.

Example	Property of Equality
$5 = 5$ $5 + 1 = 5 + 1$ $6 = 6 \quad \checkmark$	We add 1 to both sides and get $6 = 6$. So addition works! Try it for other numbers. This is called the Additive Property of Equality , since we are adding the same thing to both sides of the equation.
$5 = 5$ $5 - 3 = 5 - 3$ $2 = 2 \quad \checkmark$	If we subtract 3 from both sides, it works, too! This is called the Subtraction Property of Equality , since we are subtracting the same thing from both sides of the equation.
$5 = 5$ $5 \times 4 = 5 \times 4$ $20 = 20 \quad \checkmark$	If we multiply both sides by 4, it works, too! This is called the Multiplicative Property of Equality , since we are multiplying the same thing to both sides of the equation.
$6 = 6$ $6 \div 2 = 6 \div 2$ $3 = 3 \quad \checkmark$	If we divide both sides by 2, it works too! This is called the Division Property of Equality , since we are dividing the same thing on both sides of the equation.

Our justification is the evidence (rule/property/theorem) that allows us to complete a particular step.

Ex. Each example below uses a different Property of Equality to justify the steps needed to solve the equation.

$x - 3 = 12$ $x - 3 + 3 = 12 + 3$ $x = 15$	Add 3 to both sides of the equation because 3 is subtracted from x and addition and subtraction are inverse operations.
$x + 3 = 7$ $x + 3 - 3 = 7 - 3$ $x = 4$	Subtract 3 from both sides of the equation because 3 is added to x and addition and subtraction are inverse operations.
$\frac{x}{5} = 6$ $5 \cdot \frac{x}{5} = 5 \cdot 6$ $x = 30$	Multiply both sides of the equation by 5 because x is divided by 5 and multiplication and division are inverse operations.
$5x = 35$ $\frac{5x}{5} = \frac{35}{5}$ $x = 7$	Divide both sides of the equation by 5 because x is multiplied by 5 and multiplication and division are inverse operations.

Video help: <http://virtualnerd.com/middle-math/equations-functions/solving-subtraction/addition-property-equality-definition>

<http://virtualnerd.com/middle-math/equations-functions/solving-addition/subtraction-property-equality-definition>

<http://virtualnerd.com/middle-math/equations-functions/solving-multiplication/multiplication-property-equality-definition>

<http://virtualnerd.com/middle-math/equations-functions/solving-division/division-property-equality-definition>

<http://viewpure.com/ku96uTQsPrA?start=0&end=0>

2.7H Ready, Set, Go!

Ready

Topic: Finding an appropriate viewing window

The viewing window will only show you PART of the graph. This allows you to analyze a particular portion of your graph.

You need to understand how the viewing window is set up and how to control its size.

```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=
    
```

The WINDOW key allows you to control the viewing window.

The default window (also obtained with ZOOM #6 ZStandard) is the 10 x 10 window shown at the left.

Xmin = minimum value displayed on the x-axis
 Xmax = maximum value displayed on the x-axis
 Xscl = distance between the tick marks on the x-axis
 Ymin = minimum value displayed on the y-axis
 Ymax = maximum value displayed on the y-axis
 Yscl = distance between the tick marks on the y-axis
 Xres = pixel resolution. Xres = 1 means that functions are evaluated and graphed at every pixel on the x-axis.

Xmin must be less than Xmax

Ymin must be less than Ymax

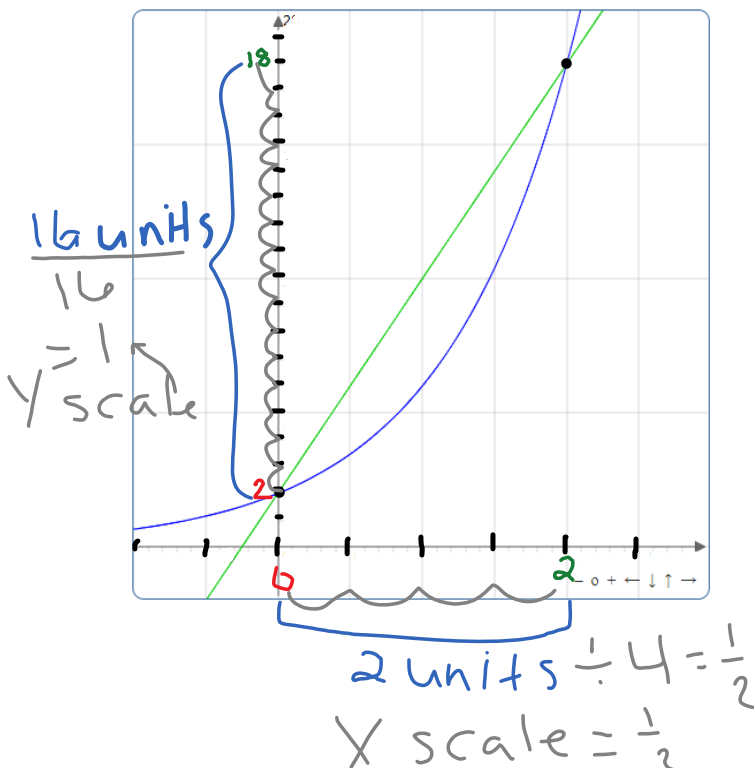
Setting
 set Xscl=0 and Yscl=0
 will turn the tick marks off.

To change window settings on the WINDOW screen:

- use your up or down arrows to move to the item to be changed
- enter the new value (which clears the original value)
- arrow down or hit ENTER.

To determine a viewing window that would allow us to see the points of intersection of the exponential function and its secant line, you can make a table of values.

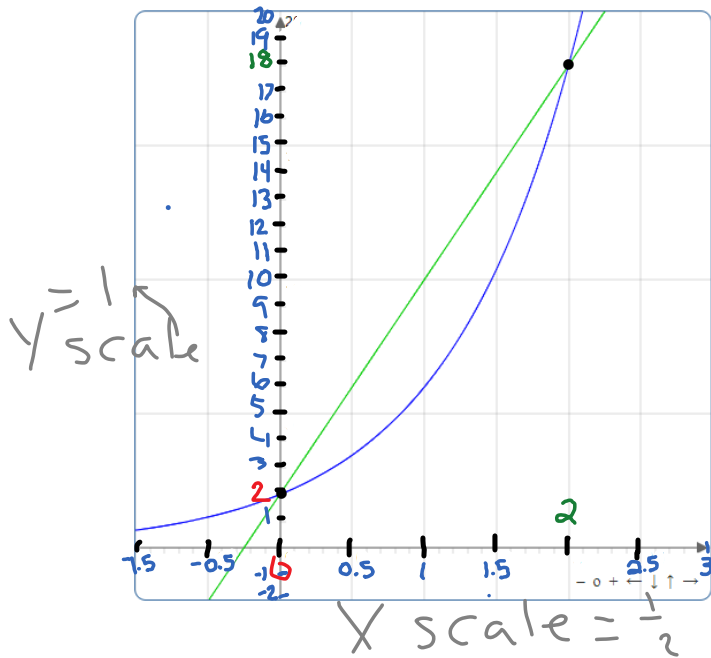
Ex. $y_1 = 2 \cdot 3^x$ and $y_2 = 8x + 2$



x	$y_1 = 2 \cdot 3^x$	$y_2 = 8x + 2$
0	2	2
1	6	10
2	18	18

On the table you want to look at values where the two functions are equal. In this table we can see that at $x = 0$ both functions have a y value of 2. At $x = 2$, both functions have a y value of 18. We can label the x and y values for these two functions. We can then find the distance between the 2 x values and divide by the number of tick marks to determine the X scale. We will repeat the process with the y axis to determine the Y scale.

Ex. $y_1 = 2 \cdot 3^x$ and $y_2 = 8x + 2$



Now that we know the X scale is $\frac{1}{2}$ or 0.5, we can label the marks on the X-axis to find the X min and X max. We will repeat on the Y-axis using the Y scale.

On the x-axis, our smallest value is the X min. For this graph, it's -1.5. The largest value on the x-axis is 3, so our X max is 3.

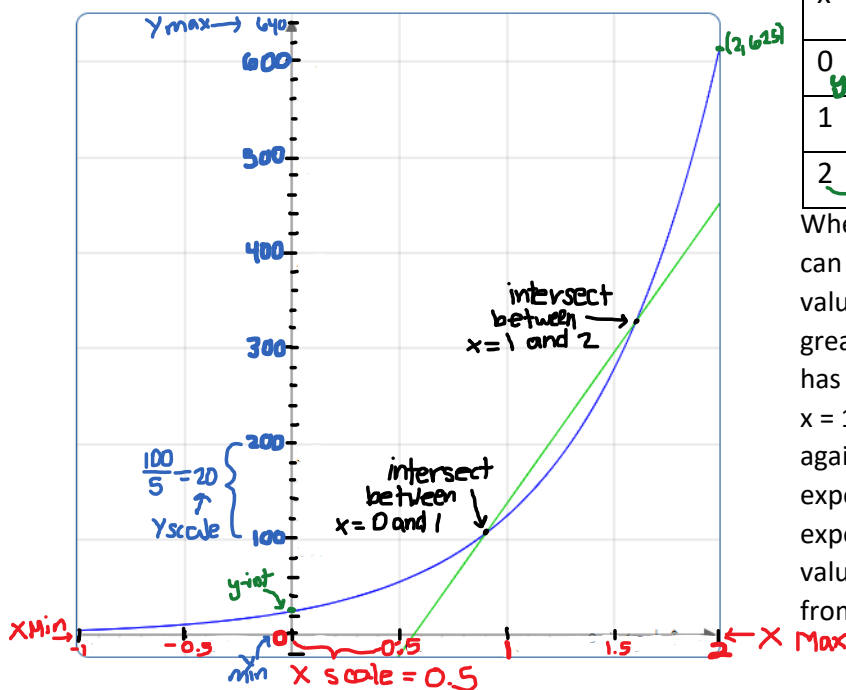
On the y-axis, our smallest value is the Y min. For this graph, it's -2. The largest value on the y-axis is 20, so our Y max is 20.

So an appropriate viewing window for our functions has the following parameters:

- WINDOW
- a. X min = -1.5
 - b. X max = 3
 - c. X scl = 0.5
 - d. Y min = -2
 - e. Y max = 20
 - f. Y scl = 1

It's not always easy to identify where the exponential function and its secant intersect using a table.

$y_1 = 25(5)^x$ and $y_2 = 317x - 179$



x	$y_1 = 25(5)^x$	$y_2 = 317x - 179$
0	25	-179
1	125	138
2	625	455

When we can't find a nice point of intersection, we can identify which function is larger for a given x value. When $x = 0$, the exponential function has a greater value. When $x = 1$, the exponential function has a smaller value. This means that between $x = 0$ & $x = 1$ the functions must intersect. This happens again between $x = 1$ and $x = 2$, at $x = 1$ the exponential function is smaller and at $x = 2$, the exponential function is larger. We can use these values to label the x-axis. We can use the y values from the table to label the y-axis.

Set

Topic: Using slope to compare change in linear and exponential models

Linear functions have a constant rate of change (slope). When you have different representation, you will need to use different methods to find the rate of change.

Example 1 Find the Average Rate of Change from a Table

The table below defines the relationship $y = f(x)$.

x	0	2	4	5
$f(x)$	26	17	5	1

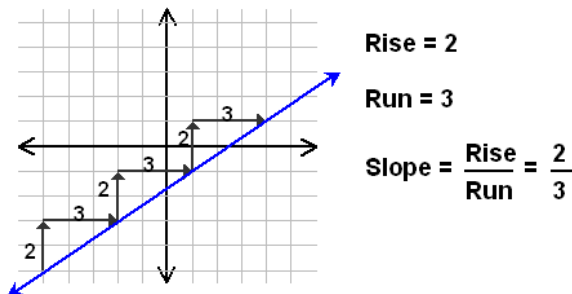
Use this table to compute the average rates of change below.

a. Find the average rate of change of f with respect to x over $[0, 4]$.

Solution Apply the definition of the average rate of change to give

$$\begin{aligned} \text{Average rate of change of } f &= \frac{\text{Change in } f}{\text{Change in } x} \\ \text{with respect to } x \text{ over } [0, 4] &= \frac{f(4) - f(0)}{4 - 0} && \text{Find the change in the numerator} \\ &= \frac{5 - 26}{4 - 0} && \text{and denominator over } [0, 4] \\ &= \frac{-21}{4} && \text{Put in the function values from} \\ & && \text{the table} \end{aligned}$$

Example 2 Find the Average Rate of Change from a Graph

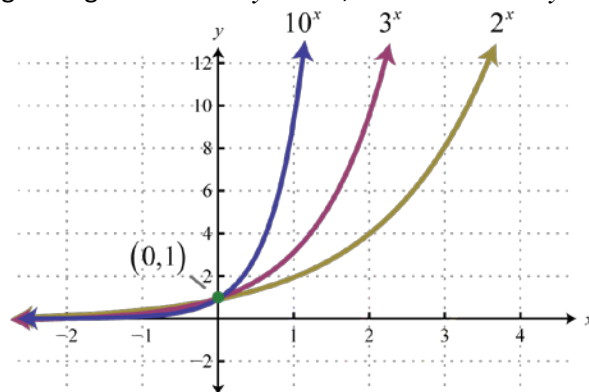


Exponential functions do not have a constant rate of change, instead they have a common ratio.

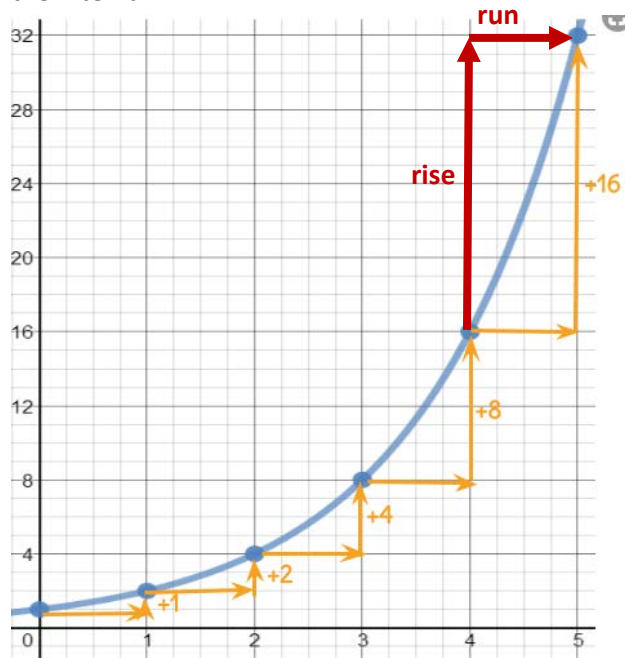
Example: Given graphs of exponential functions

When you compare graphs of exponential functions, you can compare the “steepness” of the line. The curve that turns upward more sharply is growing faster. In the graph below the function $y = 10^x$ is growing fastest.

The graph of $y = 3^x$ is growing slower than $y = 10^x$, but faster than $y = 2^x$. The graph of $y = 2^x$ is growing the slowest.

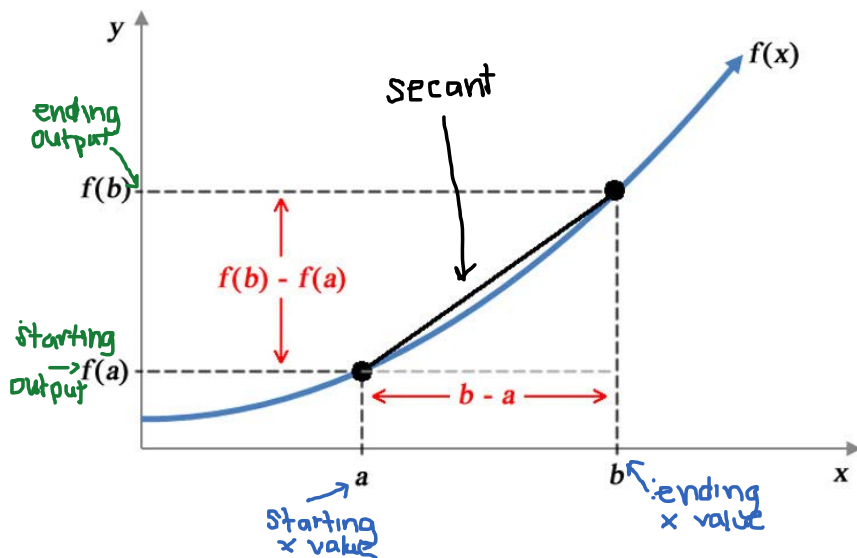


You can calculate the average rate of change for an interval by drawing a slope triangle from the endpoints of the interval.



If I want to calculate the average rate of change for the function to the left, I will need to determine the interval I want to observe. For example, the average rate of change over the interval $[4,5]$ can be modeled by the triangle to the left. For this interval the rise is 16 and the run is 1. So the average rate of change would be $\frac{16}{1}$ or 16. For a line, the average rate of change stays the same. For an exponential, the average rate of change will be different for different intervals.

To find the average rate of change for an exponential function, we will need to decide on an interval to analyze. The interval will tell us our starting and ending x values. If we draw the line connecting the starting and ending points of our interval, we will have the secant line.



We can compare the average rate of change for linear and exponential functions by breaking the table into intervals.

Ex. For the tables below, find the slope between each two points.

$\text{Slope} = \frac{\text{rise}}{\text{run}}$

x	y
1	10
2	210
3	410
4	610
5	810

run = 1, rise = 200 $m = \frac{200}{1} = 200$
 run = 1, rise = 200 $m = 200$
 run = 1, rise = 200 $m = 200$
 run = 1, rise = 200 $m = 200$

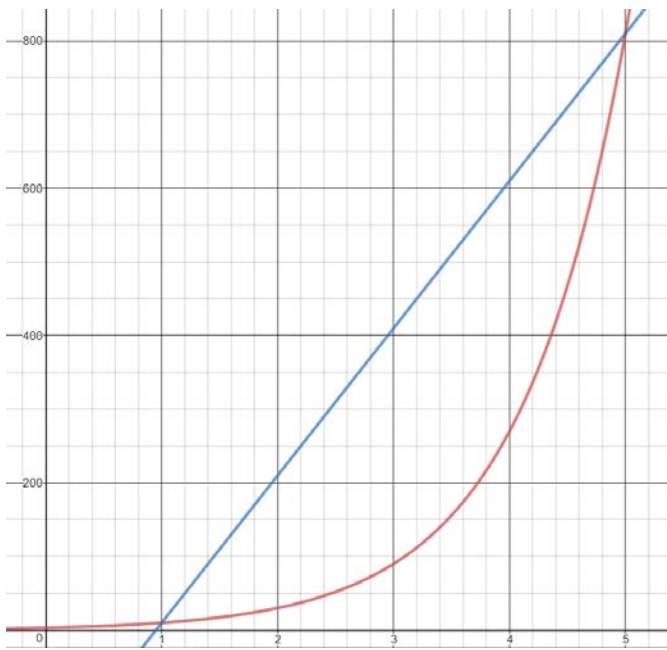
x	y
1	10
2	30
3	90
4	270
5	810

run = 1, rise = 20 $m = \frac{20}{1} = 20$
 run = 1, rise = 60 $m = 60$
 run = 1, rise = 180 $m = 180$
 run = 1, rise = 540 $m = 540$

For the linear function, the average rate of change (slope) is constant. For the exponential function, the rate of change between each pair of points is not the same. To find the average rate of change for the exponential function over the interval [1, 5], we can take the slope we got for each pair of points and average them. We will add up all of the slopes and divide by how many we have:

$$\frac{20 + 60 + 180 + 540}{4} = \frac{800}{4} = 200$$

The number we get when we take the average is the same as the slope of the linear function. This happens because both the linear and exponential function share 2 points. The line will move from the point (1, 10) up 200 and right 1 until it passes through the point (5, 810). The exponential function will go through the point (1, 10) and then curve up to the point (5, 810). The secant line is the line through the points (1, 10) and (5, 810). The secant line for an interval will give us the average rate of change for the exponential function.



You can find the average rate of change using the slope formula by plugging the x values from your given interval into the function.

$$\text{Average Rate of Change} = \frac{f(b) - f(a)}{b - a}$$

Interval: [a, b] where a and b are the x-values

f(b) is the output when you plug b into the function

f(a) is the output when you plug a into the function

For the previous example, a = 1 and f(a) = 10, b = 5 and f(b) = 810.

$$\text{Average Rate of Change} = \frac{810 - 10}{5 - 1} = \frac{800}{4} = 200$$

Video help: <http://viewpure.com/iwTu4fHoyZ0?start=0&end=0>

<http://viewpure.com/f4MYCepzLyQ?start=0&end=0>

Go

Topic: Finding the slope between two points

When you are given two points and asked to find the slope (average rate of change), you will need to use the slope formula.

Slope = $\frac{y_2 - y_1}{x_2 - x_1}$ In order to use this formula, you will need to label your two ordered pairs. The first point listed will be

your (x_1, y_1) and the second point listed will be your (x_2, y_2) . You will then subtract to find the change in y values (rise) over the change in x values (run).

Ex. $(2, 3)$ and $(10, -7)$ Once we label the two ordered pairs, we can plug them into the equation:

x_1 y_1 x_2 y_2

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 3}{10 - 2} = \frac{-10}{8} = \frac{-5}{4}$$

***Be sure to subtract in the same order (point 2 – point 1) on the top and bottom of the fraction.

***Be sure to subtract both Y values in the numerator and both X values in the denominator.

If you have points that are not integers, you may use a calculator to assist you with subtracting and simplify the formula.

Video help: <http://viewpure.com/iVvGbDr-7lQ?start=0&end=0>

<http://viewpure.com/x6qIPml2xRI?start=0&end=0> (start at 4:10)