# Laws of Logarithms

Power Rule: $\log_a m^x = x \log_a m$

Ex:
1) $\log c^4 = 4 \log c$
2) $-7 \ln x = \ln x^{-7}$

Product Rule: $\log_a (mn) = \log_a m + \log_a n$

Ex:
1) $\ln(4p) = \ln 4 + \ln p$
2) $\log_3 5 + \log_3 y = \log_3 (5y)$

Quotient Rule: $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

Ex:
1) $\log_7 \left(\frac{5}{x}\right) = \log_7 5 - \log_7 x$
2) $\ln 3 - \ln 8 = \ln \left(\frac{3}{8}\right)$

Use the laws of logarithms to rewrite the expression with no logarithms of a power, quotient or product. (Expand)

1) $\log_2(x(x-1)) = \log_2 x + \log_2(x-1)$

2) $\log_3(xy^5) = \log_3 x + \log_3 y^5$
$= \log_3 x + 5\log_3 y$

3) $\ln\left(\frac{x^3y^4}{z^6}\right) = \ln x^3 + \ln y^4 - \ln z^6$
$= 3\ln x + 4\ln y - 6\ln z$

(+) if in the numerator
(−) if in the denominator

Rewrite the expression as a single logarithm (condense)

1) $\log_3 5 + 5\log_3 2 = \log_3 5 + \log_3 2^5$
$= \log_3(5 \cdot 2^5)$
$= \log_3 160$

2) $\log_2 A + 2\log_2 B - 4\log_2 C = \log_2 A + \log_2 B^2 - \log_2 C^4$
$= \log_2\left(\frac{AB^2}{C^4}\right)$

3) $4\log x - \frac{1}{3}\log(x^2+1) + 2\log(x-1)$
$\log x^4 - \log(x^2+1)^{1/3} + \log(x-1)^2$
$\log\left(\frac{x^4(x-1)^2}{(x^2+1)^{1/3}}\right)$

4) $\ln 5 + 2\ln x + 3\ln(x^2+5)$

$\ln 5 + \ln x^2 + \ln(x^2+5)^3$

$\ln(5x^2(x^2+5)^3)$

5) $\frac{1}{3}\log(2x+1) + \frac{1}{2}[\log(x-4) - \log(x^4-x^2-1)]$

$\frac{1}{3}\log(2x+1) + \frac{1}{2}\log\left(\frac{x-4}{x^4-x^2-1}\right)$

$\log(2x+1)^{\frac{1}{3}} + \log\left(\frac{x-4}{x^4-x^2-1}\right)^{\frac{1}{2}}$

$\log\left((2x+1)^{\frac{1}{3}}\left(\frac{x-4}{x^4-x^2-1}\right)^{\frac{1}{2}}\right)$