Laws of Logarithms

Let $x = \log_b m$ and $y = \log_b n$

Using their exponential form:

$$b^x = m \quad \text{and} \quad b^y = n$$

multiplication gives:

$$m \cdot n = b^x \cdot b^y$$
$$m \cdot n = b^{x+y}$$

Changing back to log form

$$\log_b(mn) = x + y$$

Using substitution:

$$\log_b(mn) = \log_b m + \log_b n$$

Laws:

If m and n are real #'s and b is a positive # other than 1, then:

1) $\log_b(mn) = \log_b m + \log_b n$

2) $\log_b(m/n) = \log_b m - \log_b n$

3) $\log_b m^k = k \log_b m$

Express as a single logarithm

1) $\log_b m + \log_b n - 3\log_b p$

$\log_b m + \log_b n - \log_b p^3$

$\log_b \left(\frac{mn}{p^3}\right)$

2) $\ln(a+b) + \ln(a-b) - 2\ln c$

$\ln \frac{(a+b)(a-b)}{c^2} \rightarrow \ln\left(\frac{a^2-b^2}{c^2}\right)$

3) $\ln 5 + 2\ln x + 3\ln(x^2+5)$

$\ln\left(5x^2(x^2+5)^3\right)$

4) $\frac{1}{3}\log(2x+1) + \frac{1}{2}[\log(x-4) - \log(x^4-x^2-1)]$

$\log(2x+1)^{\frac{1}{3}} + [\log(x-4) - \log(x^4-x^2-1)]^{\frac{1}{2}}$

$\log(2x+1)^{\frac{1}{3}} + \log\left(\frac{x-4}{x^4-x^2-1}\right)^{\frac{1}{2}}$

$\log\left((2x+1)^{\frac{1}{3}}\left(\frac{x-4}{x^4-x^2-1}\right)^{\frac{1}{2}}\right)$

Rewrite in a form with no logarithms of a product, quotient, root, or power.

1) $\log_5 (\sqrt[3]{x^2+1})^{\frac{1}{3}}$

$\frac{1}{3}\log_5 (x^2+1)$

2) $\log\left(\frac{x^3y^4}{z^6}\right)$

$3\log x + 4\log y - 6\log z$

3) $\log_5 \sqrt{\frac{x-1}{x+1}}$

$\log_5 \left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$

$\frac{1}{2}\log_5\left(\frac{x-1}{x+1}\right)$

$\frac{1}{2}\left[\log_5 (x-1) - \log_5 (x+1)\right]$

4) $\ln\left(x\sqrt{\frac{y}{z}}\right)$

$\ln x + \ln\left(\frac{y}{z}\right)^{\frac{1}{2}}$

$\ln x + \frac{1}{2}\ln\frac{y}{z}$

$\ln x + \frac{1}{2}\left(\ln y - \ln z\right)$

5) $\ln\left(\frac{x^3\sqrt{x-1}}{3x+4}\right)$

$3\ln x + \frac{1}{2}\ln(x-1) - \ln(3x+4)$