Inverse Functions

A function that reverses another function

- A reflection in the line $y=x$
- All points (x,y) become (y,x)
- Notation $f^{-1}(x)$

$f(x)=2x+7$

Domain: $(-\infty,+\infty)$

Range: $(-\infty,+\infty)$

$x=2y+7$

$x-7=2y$

$\frac{1}{2}x-\frac{7}{2}=y$

$f^{-1}(x)=\frac{1}{2}x-\frac{7}{2}$

$f(x)=2x+7$

$f(1)=2(1)+7$

$f(1)=9$

$f^{-1}(9)=1$

$f^{-1}(9)=\frac{1}{2}(9)-\frac{7}{2}$

$=\frac{9}{2}-\frac{7}{2}$

$=1$

Find each of the following given:

$$g(3)=7 \quad \text{and} \quad g(5)=-2$$

1) $g^{-1}(7)=3$

2) $g^{-1}(-2)=5$

3) $g^{-1}(\overset{7}{g(3)})=3$

4) $g(\overset{5}{g^{-1}(-2)})=-2$

Two functions are inverses of each other if:

- $g(f(x))=x$ for all x's in the domain of f.
- $f(g(x))=x$ for all x's in the domain of g.

Show f(x) and g(x) are inverses.

$$f(x)=2x+7 \qquad g(x)=\frac{x-7}{2}$$

$$f(g(x))=2\left(\frac{x-7}{2}\right)+7$$
$$=x-7+7$$
$$=x$$

$$g(f(x))=\frac{2x+7-7}{2}$$
$$=\frac{2x}{2}$$
$$=x$$

f(x) and g(x) are inverses because

$$f(g(x))=g(f(x))=x$$

Given each function, find its inverse and the domain of the inverse.

1) $y = \sqrt{x+2}$

Domain: $[-2, +\infty)$

Range: $[0, +\infty)$

$x = \sqrt{y+2}$

$x^2 = y+2$

$x^2 - 2 = y$

$f^{-1}(x) = x^2 - 2$

Domain: $[0, +\infty)$

2) $y = x^2 + 6x$

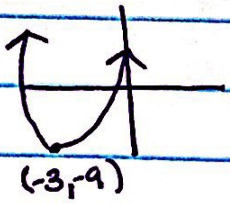
$y = (x^2 + 6x + \underline{\ 9\ }) - \underline{\ 9\ }$

$y = (x+3)^2 - 9$

Restrict $\longrightarrow$ $[-3, +\infty)$

Domain: $(-\infty, +\infty)$

Range: $[-9, +\infty)$



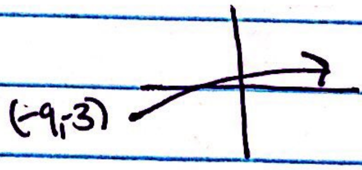
$x = (y+3)^2 - 9$

$x + 9 = (y+3)^2$

$\sqrt{x+9} = y + 3$

$\sqrt{x+9} - 3 = y$

$f^{-1}(x) = \sqrt{x+9} - 3$

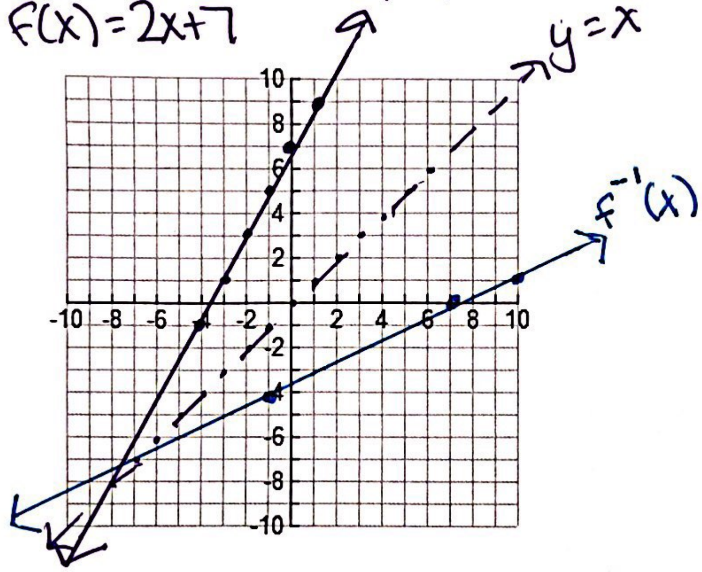


Domain: $[-9, +\infty)$

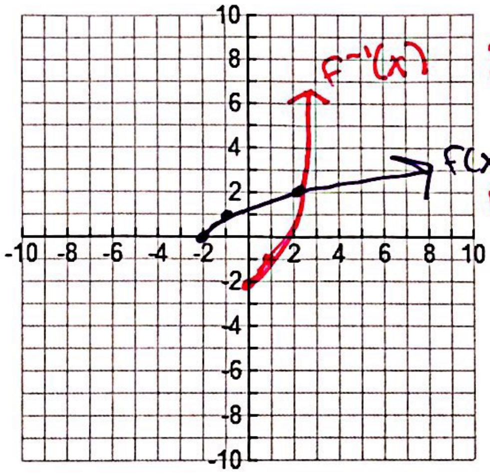
Range: $[-3, +\infty)$

The inverse is a function only if the function is 1 to 1.

$F(x)=2x+7$



$y=\sqrt{x+2}$ Domain: $[-2,+\infty)$ Range: $[0,+\infty)$



Domain: $[0,+\infty)$

Range: $[-2,+\infty)$