Sum of an Infinite Geometric Series

If half of a pizza is left and you keep taking ½ of what is left over and over, eventually, the sum of the pizza that has been taken will approach 1.

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$ approaches 1. Even though you continue to add more and more terms, the sum will never go beyond 1 because the amount you are adding is less than 1.

In a geometric series, if $|r|<1$, the sum of an infinite amount of terms is $S_\infty = \frac{a}{1-r}$.

(If if $|r|$ is not less than 1, then there is no infinite sum ... the sum will keep getting bigger and bigger. An infinite sum in that case would not exist.)

Example:

1. Find the sum (if it exists) of: $2+\frac{2}{5}+\frac{2}{25}+\frac{2}{125}+\cdots$

$r=\frac{1}{5}$

$$S=\frac{2}{1-\left(\frac{1}{5}\right)}=\frac{2}{\frac{4}{5}}=\frac{5}{2}$$

Convergent: when an infinite sum can be taken $|r|<1$

2. Find the sum (if it exists) of: $\frac{3}{4}+3+12+48+\cdots$

$r=4$

$r>1$
so no infinite sum

divergent: when an infinite sum can't be taken. $|r|>1$

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Infinite Geometric Sums & Sigma Notation

Name ____________________

Decide whether the infinite series has a sum.

1. $10 + 20 + 40 + \ldots$

$r=2$ Divergent

2. $3 + \frac{12}{5} + \frac{48}{25} + \ldots$ Convergent

$r=\frac{4}{5}$

3. $\sum_{k=1}^{\infty} 3(\frac{7}{2})^{k-1}$

$r=\frac{7}{2}$ Divergent

4. $\sum_{k=1}^{\infty} -4(\frac{1}{6})^{k-1}$

$r=\frac{1}{6}$ Convergent

5. $\sum_{k=1}^{\infty} 5(-\frac{2}{5})^{k-1}$

$r=\frac{2}{5}$ Convergent

6. $\sum_{k=1}^{\infty} (\frac{8}{7})^{k-1}$

$r=\frac{8}{7}$ divergent

Find the sum of the series (if it has one).

1. $\sum_{k=1}^{\infty} 2(\frac{2}{3})^{k-1}$

$r=\frac{2}{3}$

$S=\frac{2}{1-\frac{2}{3}}$

$=\frac{2}{\frac{1}{3}}$

$=6$

2. $\sum_{k=1}^{\infty} 12(\frac{1}{4})^{k-1}$

$r=\frac{1}{4}$

$S=\frac{12}{1-\frac{1}{4}}$

$=\frac{12}{\frac{3}{4}}$

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3. $\sum_{k=1}^{\infty} 5(\frac{4}{9})^{k-1}$

$S=\frac{5}{1-\frac{4}{9}}$

$=\frac{5}{\frac{5}{9}}$

$=9$

4. $\sum_{k=1}^{\infty} \frac{1}{8}(8)^{k-1}$ Divergent

$r=8$

5. $\sum_{k=1}^{\infty} 10(-\frac{1}{2})^{k-1}$

$S=\frac{10}{1-(-\frac{1}{2})}$

$=\frac{10}{\frac{3}{2}}$

$=\frac{20}{3}$

6. $\sum_{k=1}^{\infty} -5(0.1)^{k-1}$

$S=\frac{-5}{1-.1}$

$=\frac{-5}{.9}$

$=-\frac{50}{9}$

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