12.3 Geometric Sequences

Date _____

Geometric sequence – sequence in which each term is multiplied by a common ratio that is a _____ to get the next term

$a,\ ar,\ ar^2,\ ar^3,\ ar^4,\ \ldots.$

2, 4, 8, 16 (×2 ×2 ×2)

a: First Term r: common ratio $r = \frac{a_2}{a_1}$

nth term: $a_n = ar^{n-1}$

Examples:

Find r and the next two terms of the geometric sequence.

1. 4, 2, 1, ... $\frac{1}{2}, \frac{1}{4}$

   r = $\frac{1}{2}$ $r = \frac{2}{4} = \frac{1}{2}$

2. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots$ $-\frac{1}{16}, \frac{1}{32}$

   r = $-\frac{1}{2}$ $r = \frac{-\frac{1}{4}}{\frac{1}{2}}$

3. Find the next 3 terms in the sequence.

   3, -6, 12, -24, ... 48, -96, 192 (×-2 ×-2)

4. Find the nth term

   a =3, r = 2, $a_n = 3(2)^{n-1}$ $a_n = ar^{n-1}$

5. Find the 7th term of

   -1, -4, -16, ...

   $r = \frac{-4}{-1} = 4$

   $a_n = ar^{n-1}$

   $a_n = (-1)(4)^{n-1}$

   $a_7 = (-1)(4)^{7-1}$

   $a_7 = (-1)(4)^6$

   $a_7 = -4096$

6. 2, −10, 50, −250, 1250, ...

   a = 2, r = −5, $a_n = 2(-5)^{n-1}$, $a_{10} = -3906250$

   $a_{10} = 2(-5)^{10-1}$

   $= 2(-5)^9$

7. Find the 1st term given $a_8 = 64$, $r = 2$.

$a_n = ar^{n-1}$ (8 ↑, a_n ↑)

$64 = a(2)^{8-1}$

$64 = a(2)^7$

$64 = a(128)$

$a = \frac{1}{2}$

8. Find the 7th term if $a_3 = 96$ & $r = 4$.

$a_n = ar^{n-1}$ (n ↑, a_n ↑)

$96 = a(4)^{3-1}$

$96 = a(4)^2$

$96 = a(16)$

$6 = a$

$a_n = 6(4)^{n-1}$

$a_7 = 6(4)^{7-1}$

$a_7 = 6(4)^6$

$a_7 = 24576$

9. $a_3 = \frac{63}{4}$, $a_6 = \frac{1701}{32}$, $a_5 = ?$ (n ↑, a_n ↑, n ↑, a_n ↑)

$a_n = ar^{n-1}$

$\frac{63}{4} = ar^{3-1}$

$\frac{63}{4} = ar^2$

$\frac{63}{4r^2} = a$

$\frac{1701}{32} = ar^{6-1}$

$\frac{1701}{32} = ar^5$

← Substitution

$\frac{1701}{32} = \left(\frac{63}{4r^2}\right) r^5$

$\frac{1701}{32} = \frac{63r^3}{4}$

$6804 = 2016r^3$

$\frac{27}{8} = r^3$

$\frac{3}{2} = r$ → so

$a = \frac{63}{4r^2}$

$= \frac{63}{4\left(\frac{3}{2}\right)^2}$

$= \frac{63}{9}$

$a = 7$

$a_n = 7\left(\frac{3}{2}\right)^{n-1}$

$a_5 = 7\left(\frac{3}{2}\right)^{5-1}$

$= 7\left(\frac{3}{2}\right)^4$

$a_5 = \frac{567}{16}$

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