Module 8 Extra Test Review

1. The 14th term of an arithmetic sequence is 39.7 and the 3rd term is 6.7. Find the common difference and the 32nd term for this sequence.

$a_{14} = 39.7$ $6.7 = a + 2d$ $a + 2(3) = 6.7$ $d = \underline{3}$
$a_3 = 6.7$ $-(39.7 = a + 13d)$ $a = .7$ $a_{32} = \underline{93.7}$
$\dfrac{-33 = -11d}{3 = d}$ $a_{32} = .7 + (32-1)(3)$

2. Find the sum of the arithmetic series $3 + 5 + 7 + \cdots + 51$.

$S_n = \dfrac{n}{2}(a + a_n)$ $51 = 3 + (n-1)2$ $S_n = \underline{675}$
$S_{25} = \dfrac{25}{2}(3 + 51)$ $51 = 3 + 2n - 2$
$51 = 2n + 1$
$50 = 2n$ $n = 25$

3. Which term of the arithmetic sequence $-1.8, -1.6, -1.4, \ldots$ is 0.8?

$d = -1.6 - (-1.8)$ $a_n = a + (n-1)d$ $2n = 2.8$ $n = \underline{14}$
$d = .2$ $.8 = -1.8 + (n-1)(.2)$ $n = 14$
$.8 = -1.8 + 2n - .2$
$.8 = -3.8 + 2n$ (×2)

4. Find the first four terms and the 15th term of the sequence: $a_n = \dfrac{-1}{3n+4}$

$a_1 = \underline{\dfrac{-1}{7}}$ $\dfrac{-1}{3(1)+4}$ $a_2 = \underline{\dfrac{-1}{10}}$ $\dfrac{-1}{3(2)+4}$ $a_{15} = \underline{\dfrac{-1}{49}}$ $\dfrac{-1}{3(15)+4}$

$a_3 = \underline{\dfrac{-1}{13}}$ $\dfrac{-1}{3(3)+4}$ $a_4 = \underline{\dfrac{-1}{16}}$ $\dfrac{-1}{3(4)+4}$

5. Write out the sum without using sigma notation:

$2(0)-3 + 2(1)-3 + 2(2)-3 + 2(3)-3$ $\displaystyle\sum_{j=0}^{4} 2j - 3$
$-3 + (-1) + 1 + 3$
$+2(4)-3$
$+5$ $\underline{5}$

6. Find the sum of the infinite geometric series $3 + \dfrac{3}{8} + \dfrac{3}{64} + \dfrac{3}{512} + \cdots$

$S = \dfrac{a}{1-r}$ $r = \dfrac{\frac{3}{8}}{3} = \dfrac{1}{8}$ $\dfrac{24}{7}$

$= \dfrac{3}{1-\frac{1}{8}}$ $S_\infty = $

$= \dfrac{3}{\frac{7}{8}}$ 11

7. The third term of the geometric sequence is 2 and the 9th term is $\frac{1}{2048}$. Find the common ratio and the first term of this sequence.

$a_3 = 2$

$a_9 = \frac{1}{2048}$

$a_n = ar^{n-1}$

$2 = ar^{3-1}$

$2 = ar^2$

$\frac{1}{2048} = ar^{9-1}$

$\frac{1}{2048} = ar^8$

$\frac{2}{r^2} = a$

$\frac{1}{2048} = \frac{2}{r^2} \cdot r^8$

$\frac{1}{2048} = 2r^6$

$\frac{1}{4096} = r^6$

$\sqrt[6]{\frac{1}{4096}} = r$

$r = 4$

$r = \underline{4}$

$a_1 = \underline{1/8}$

$a = \frac{2}{4^2}$

8. The first term of the geometric sequence is $\frac{1}{2}$ and the second term is 1. Find the sixth term.

$\frac{1}{2}, 1$

$\cdot 2$

$a_n = ar^{n-1}$

$a_6 = \frac{1}{2}(2)^{6-1}$

$a_6 = \frac{1}{2}(2)$

$a_6 = \underline{16}$

9. How many terms are in the sequence $0.75, 1.5, 3, \ldots, 3145728$?

$r = \frac{1.5}{.75} = 2$

$3145728 = .75(2)^{n-1}$

$4194304 = 2^{n-1}$

$2^{22} = 2^{n-1}$

$n - 1 = 22$

$n = \underline{23}$

10. A Kleenex display has 48 rows of Kleenex boxes. There are 7 on the top row, 13 in the second row, 19 in the third row and so on. How many boxes are in row 28? How many total boxes are in the display?

$n = 48$

$7, 13, 19, \ldots$

$+6$

$a_{28} = 7 + (28-1)(6)$

$a_{28} = 169$

$a_{28} = \underline{169}$

$S_n = \underline{7104}$

$S_{48} = \frac{48}{2}(7 + 289)$

$a_{48} = 7 + (48-1)(6)$

$a_{48} = 289$

12