Applications of Logs

Compound Interest

$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ Compandancy

P - principle
r - rate (decimal)
n - # of times compounding
t - time (years)

$A(t) = Pe^{rt}$ Continuously

Population Growth

$N(t) = N_0 e^{rt}$

N_0 = initial population

Half-Life

$A(t) = A_0\left(\frac{1}{2}\right)^{t/k}$

A_0 = initial amount
t - time
k - length of half life

1) The half life of Carbon-11 is 20 minutes. How long will it take for 800g of Carbon-11 to decay to 640?

$$A(t) = A_0\left(\frac{1}{2}\right)^{t/K}$$

$$A(t) = A_0\left(\frac{1}{2}\right)^{t/20}$$

$$640 = 800\left(\frac{1}{2}\right)^{t/20}$$

$$.8 = \left(\frac{1}{2}\right)^{t/20}$$

$$\log_{.5} .8 = \frac{t}{20}$$

$$\frac{\log .8}{\log .5} = \frac{t}{20}$$

$$20\left(\frac{\log .8}{\log .5}\right) = t$$

$$6.44\text{min} = t$$

2) Suppose you invest P dollars at 3% compounded continuously. How long will it take for your money to grow by ~~$800~~? 50%?

$$A(t) = Pe^{rt}$$

$$1.5 = 1e^{.03t}$$

$$1.5 = e^{.03t}$$

$$\ln 1.5 = .03t$$

$$\frac{\ln 1.5}{.03} = t$$

$$13.5\text{yrs} = t$$