

* Expected Value of a fair game is 0.
Expected Value

The projected winnings for a given game based on the probability of winning and the amount you would win.

* Kind of like a weighted average.

Example 1: A spinner has 8 slots you can land on. 2 green slots are marked winners. You have to pay \$2 to play. If you land on a green winner you will win \$5. What is the expected value?
 8 slots
 2 green

Outcome	Win	Lose
Probability	$\frac{2}{8} = \frac{1}{4}$	$\frac{6}{8} = \frac{3}{4}$
Payoff	5	0

Whose Favor?
House

Fair?
NO

payoff (probability)

$$5\left(\frac{1}{4}\right) - 0\left(\frac{3}{4}\right) - 2 = -0.75$$

↑
pay

↑
player is expected to lose \$0.75 per game

Example 2: If the sum of two rolled die is 8 or more, you win \$2, if not, you lose \$1. Is this game fair?

$$p(8 \text{ or more}) = \frac{15}{36} = \frac{5}{12}$$

$$p(\text{less than } 8) = \frac{21}{36} = \frac{7}{12}$$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$2\left(\frac{5}{12}\right) - 1\left(\frac{7}{12}\right) = \frac{1}{4}$$

↑ win ↑ Lose

\$0.25

Whose Favor?
Player

Fair?
NO

Example 3: Two coins are tossed. If both land heads up, then player A wins \$4 from player B. If exactly 1 coin lands heads up, then B wins \$1 from A. If both land tails up then B wins \$2 from A. Is this game fair?

$$4\left(\frac{1}{4}\right) - 1\left(\frac{2}{4}\right) - 2\left(\frac{1}{4}\right) = 0$$

↑ 2 heads ↑ 1 head ↑ 2 tails

Fair -

	T	H
T	TT	TH
H	HT	HH