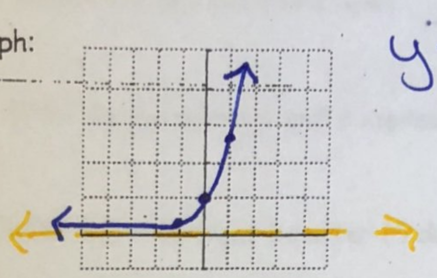
Section 6.2 - The Natural Exponential Function

Name: ______________
Date: ________ Period: ________

Definition: A function of base e.

$e \approx 2.7182818...$

Graph:



$y = e^x$

Domain: $\mathbb{R}$

Range: $y > 0$

Asymptote: $y = 0$

State the domain, range, and asymptote for the following graphs.

1. $y = -e^x$

D: $\mathbb{R}$
R: $y < 0$
A: $y = 0$

2. $y = 1 + e^x$ $\quad y = e^x + 1$

D: $\mathbb{R}$
R: $y > 1$
A: $y = 1$

3. $y = e^{x-2}$

D: $\mathbb{R}$
R: $y > 0$
A: $y = 0$

4. $y = e^{x-3} + 4$

D: $\mathbb{R}$
R: $y > 4$
A: $y = 4$

Compound Interest

Formula:

Total Amount → $A(A) = P\left(1 + \frac{r}{n}\right)^{nt}$

P = principle

r = rate (decimal)

n = # of times compounding

t = time (years)

Annually - 1
Semiannually - 2
Triannually - 3
Quarterly - 4
Monthly - 12
Weekly - 52
Daily - 365

Example: A sum of $1000 is invested at an interest rate of 12% per year. Find the amounts in the account after 5 years if interest is compounded annually, semiannually, quarterly, monthly, and daily.

Annually:
$n = 1$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = \underbrace{1000}_{\text{3rd}}\underbrace{\left(1 + \frac{.12}{1}\right)}_{\text{1st}}^{\overbrace{(1 \cdot 5)}^{\text{2nd}}}$

$A = \$1762.34$

Semiannually:

Quarterly:

P - 1000
r - .12
t - 5

Monthly:

Daily:
$n = 365$

$A = \underbrace{1000}_{\text{3rd}}\underbrace{\left(1 + \frac{.12}{365}\right)}_{\text{1st}}^{\overbrace{(365 \cdot 5)}^{\text{2nd}}}$

$A = \$1821.94$

Continuously Compounded Interest

Formula: $A(t) = Pe^{rt}$

Example: Find the amount after 5 years if \$1000 is invested at an interest rate of 12% per year, compounded continuously.

$P = 1000$ $\quad A(t) = 1000e^{(.12 \times 5)}$

$r = .12$

$t = 5$ $\quad A(t) = \$1822.12$

Exponential Growth (Population)

Formula: $N(t) = N_0 e^{rt}$

$n(t)$ = Total Population

n_0 = Initial Population

r = Rate (decimal)

t = Time

Example #1: The initial bacterium count in a culture is 500. A biologist later makes a sample count of bacteria in the culture and he finds that the relative rate of growth is 40% per hour.

a) Find a function that models the number of bacteria after t hours. $N = 500e^{(.4t)}$

b) What is the estimated count after 10 hours? $N = 500e^{(.4 \times 10)}$

t $\quad N = 27299.08$

Example #2: Under ideal conditions, a certain type of bacteria has a relative growth rate of 220% per hour. A number of these bacteria are introduced accidentally into a food product. Two hours after contamination, a bacterium count shows that there are about 40,000 bacteria in the food.

a) Find the initial number of bacteria introduced into the food.

$N = N_0 e^{rt}$

$40{,}000 = N_0 e^{(2.2 \times 2)}$ $\quad \frac{40{,}000}{e^{4.4}} = N_0$

$40{,}000 = N_0 e^{4.4}$ $\quad N_0 = 491.09$

b) Estimate the number of bacteria in the food 3 hours after contamination.

$N = 491(e)^{(2.2 \times 3)}$

$N = 36{,}0931.74$