5.1 Integral Exponents

Laws of Exponents

Same Base:

1) $b^x \cdot b^y = b^{(x+y)}$ 2) $\frac{b^x}{b^y} = b^{(x-y)}$

Same Exponent:

1) $(ab)^x = a^x b^x$ 2) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Power to a Power

1) $(b^x)^y = b^{xy}$

Negative Exponents

1) $b^{-x} = \frac{1}{b^x}$ 2) $\frac{1}{b^{-x}} = b^x$

5.2 Rational Exponents

Laws:

$b^{\frac{1}{n}} = \sqrt[n]{b}$ $\qquad$ $b^{\frac{p}{n}} = \sqrt[n]{b^p}$

Write the expression using a radical and no negative exponents.

1) $3y^{\frac{2}{5}} = 3\sqrt[5]{y^2}$

2) $a^{\frac{4}{7}} b^{\frac{-4}{7}} = \frac{a^{\frac{4}{7}}}{b^{\frac{4}{7}}}$

$= \left(\frac{a}{b}\right)^{\frac{4}{7}}$

$= \sqrt[7]{\left(\frac{a}{b}\right)^4}$

Write the expression using positive rational exponents.

1) $\sqrt[3]{8x^7} = (8x^7)^{\frac{1}{3}}$

$= 8^{\frac{1}{3}} x^{\frac{7}{3}}$

$= 2x^{\frac{7}{3}}$

Simplify

1) $\left(\frac{27}{8}\right)^{\frac{2}{3}} = \left(\frac{3}{2}\right)^{2}$ (square, cube root)

$= \frac{9}{4}$

2) $2n^{\frac{1}{3}}\left(n^{\frac{2}{3}} + n^{-\frac{1}{3}}\right)$

$2n^{\frac{3}{3}} + 2n^{0}$

$2n + 2$

3) $\frac{y^{-\frac{1}{3}} - 3y^{\frac{2}{3}}}{y^{-\frac{4}{3}}}$

$y^{\frac{4}{3}}\left(y^{-\frac{1}{3}} - 3y^{\frac{2}{3}}\right)$

$y^{\frac{3}{3}} - 3y^{\frac{6}{3}}$

$y - 3y^{2}$

Solve

1) $9^{x} = 3^{10}$

$3^{2x} = 3^{10}$

$2x = 10$

$x = 5$

2) $27^{1-x} = \left(\frac{1}{9}\right)^{2-x}$

$27^{1-x} = 9^{-(2-x)}$

$3^{3(1-x)} = 3^{-2(2-x)}$

$3(1-x) = -2(2-x)$

$3 - 3x = -4 + 2x$

$7 = 5x$

$\frac{7}{5} = x$

Factor

1) $(x-1)^{\frac{1}{2}} - x(x-1)^{-\frac{1}{2}}$

$(x-1)^{-\frac{1}{2}}\left[(x-1)^{1} - x(1)\right]$

$(x-1)^{-\frac{1}{2}}\left[(x-1) - x\right]$

$(x-1)^{-\frac{1}{2}}(-1)$

$-(x-1)^{-\frac{1}{2}}$

2) $(x+1)^{\frac{3}{2}} - 4(x+1)^{\frac{1}{2}}$

$(x+1)^{\frac{1}{2}}\left((x+1) - 4\right)$

$(x+1)^{\frac{1}{2}}(x-3)$

Solve

1) $x^{\frac{4}{3}} - 6x^{\frac{2}{3}} + 8 = 0$

$(x^{\frac{2}{3}})^2 - 6x^{\frac{2}{3}} + 8 = 0$

$(x^{\frac{2}{3}} - 4)(x^{\frac{2}{3}} - 2) = 0$

$x^{\frac{2}{3}} = 4 \quad x^{\frac{2}{3}} = 2$

$x = 8 \quad x = 2\sqrt{2}$

Substitution:

$m = x^{\frac{2}{3}}$

$m^2 - 6m + 8 = 0$

$(m-4)(m-2) = 0$

$m = 4 \quad m = 2$

$(x^{\frac{2}{3}})^{3/2} = (4)^{3/2} \quad (x^{\frac{2}{3}})^{3/2} = 2^{3/2}$

$x = 2^3 \quad x = 8^{\frac{1}{2}}$

$x = 8 \quad x = \sqrt{8}$

$x = 2\sqrt{2}$

2) $4^{2x} - 10 \cdot 4^{x} + 16 = 0$

$(4^x)^2 - 10 \cdot 4^x + 16 = 0$

$(4^x - 8)(4^x - 2) = 0$

$4^x = 8$ $\quad$ $4^x = 2$

$2^{2x} = 8$ $\quad$ $2^{2x} = 2^1$

$2^{2x} = 2^3$ $\quad$ $2x = 1$

$2x = 3$ $\quad$ $x = \frac{1}{2}$

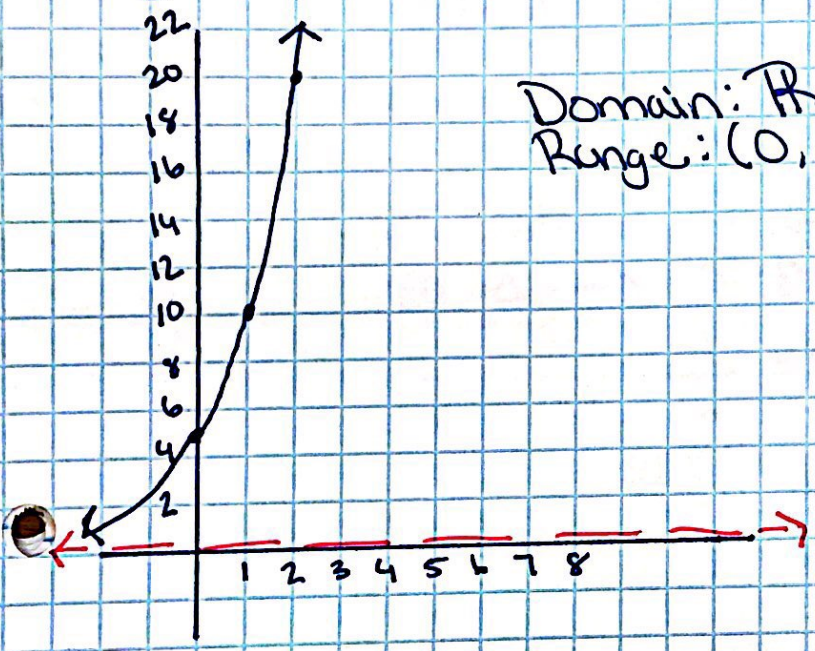
$x = \frac{3}{2}$

5.3 Exponential Functions

$f(x) = a\,b^{x} + c$

- b: rate
- c: V.S

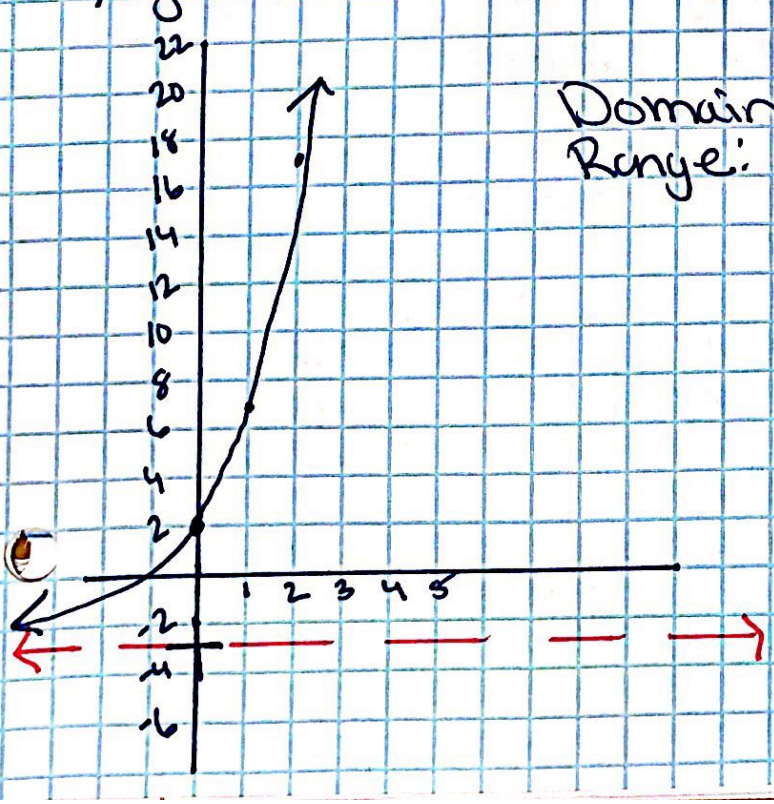
1) $y = 5(2)^{x}$



Domain: $\mathbb{R}$
Range: $(0, +\infty)$

x	y
0	$5(2)^{0} = 5$
1	$5(2)^{1} = 10$
2	$5(2)^{2} = 20$

2) $y = 5(2)^{x} - 3$



Domain: $\mathbb{R}$
Range: $(-3, +\infty)$

1) Find the equation of an exponential function given $f(0)=80$ and $f(4)=5$

$(0, 80)$ $(4, 5)$

$y = ab^x$

$80 = ab^0$ $\quad 5 = ab^4$

$80 = a \longrightarrow \quad 5 = 80b^4$

$\frac{1}{16} = b^4$

$\sqrt[4]{\frac{1}{16}} = b$

$\frac{1}{2} = b$

$y = 80\left(\frac{1}{2}\right)^x$

2) Find a function of the form $y = ab^x + c$ given the following:

$f(0)=6$ $\quad f(1)=18$ $\quad$ As $x \to -\infty$, $f(x) \to 2$

$(0, 6)$ $\quad (1, 18)$ $\quad$ HA: $y=2$

$6 = ab^0 + 2$ $\quad 18 = ab + 2$

$4 = a$ $\quad 18 = 4b + 2$

$16 = 4b$

$4 = b$

$y = 4(4)^x + 2$