

Composition of Functions

* Basically substitution

Notation - $f(g(x))$ f of g of x
 $(f \circ g)(x)$

$$f(x) = 2x + 3$$

$$g(x) = x - 5$$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x - 5) \\ &= 2(x - 5) + 3 \\ &= 2x - 10 + 3 \\ &= 2x - 7\end{aligned}$$

Finding domains of composite functions:

1) Find the domain of the "inside" (input function).
If there are any restrictions, keep them.

2) Find the composite function

3) Domain of ~~com~~ the new function.

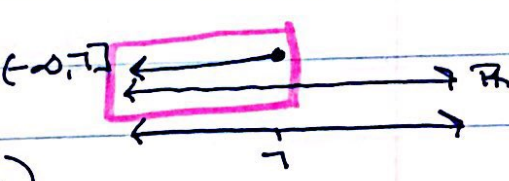
4) Domain of the composite function is
the domain of the new function
minus the restrictions from the input function.

Find the domain of $f(g(x))$ and the Composite function.

$$1) f(x) = x^2 + 2 \quad g(x) = \sqrt{7-x} \quad \begin{array}{l} 7-x \geq 0 \\ 7 \geq x \end{array}$$

$$(f \circ g)(x) = (\sqrt{7-x})^2 + 2$$

$$= 7-x+2$$

$$= 9-x \quad (-\infty, +\infty)$$


Domain of $(f \circ g)(x)$ is $(-\infty, 7]$

$$2) f(x) = \frac{3x}{x-1} \quad g(x) = \frac{2}{x} \quad \begin{array}{l} (-\infty, 0) \cup (0, +\infty) \\ x \neq 0 \end{array}$$

$$(f \circ g)(x) = \frac{3\left(\frac{2}{x}\right)}{1\left(\frac{2}{x}\right) - 1}$$

$$= \frac{\left(\frac{6}{x}\right)}{\left(\frac{2}{x}\right) - 1}$$

$$= \frac{\frac{2}{x} - \frac{1}{1}}{2-x} \quad x \neq 2$$

$$= \frac{\frac{6}{x}}{2-x}$$

Domain of $(f \circ g)(x)$ is $(-\infty, 0) \cup (0, 2) \cup (2, +\infty)$