Binomial Theorem

$(x+y)^{12}$

$(x+y)^2 = (x+y)(x+y)$
$= x^2+2xy+y^2$

$(x+y)^3 = (x+y)(x+y)(x+y)$
$= (x+y)(x^2+2xy+y^2)$
$= x^3+2x^2y+xy^2+x^2y+2xy^2+y^3$
$= x^3+3x^2y+3xy^2+y^3$

1st term - Exponents decrease
2nd term - Exponents increase

1) $(x+y)^3$ n(exponent) - 3 r - starts at 0 increases by 1

1st	${}_3C_0$	x^3y^0	$= (1)(x^3)(1)$
2nd	${}_3C_1$	x^2y^1	$= (3)(x^2)(y)$
3rd	${}_3C_2$	x^1y^2	$= (3)(x)(y^2)$
4th	${}_3C_3$	x^0y^3	$= (1)(1)(y^3)$

$x^3+3x^2y+3xy^2+y^3$

2) $(2x+5)^4$ $n=4$

1st $_4C_0\ (2x)^4(5)^0 = (1)(16x^4)(1)$

2nd $_4C_1\ (2x)^3(5)^1 = (4)(8x^3)(5)$

3rd $_4C_2\ (2x)^2(5)^2 = (6)(4x^2)(25)$

4th $_4C_3\ (2x)^1(5)^3 = (4)(2x)(125)$

5th $_4C_4\ (2x)^0(5)^4 = (1)(1)(625)$

$16x^4+160x^3+600x^2+1000x+625$

3) $(x-y)^5 = (x+(-y))^5$ $n=5$

1st $_5C_0\ (x)^5(-y)^0 = (1)(x^5)(1)$

2nd $_5C_1\ (x)^4(-y)^1 = (5)(x^4)(-y)$

3rd $_5C_2\ (x)^3(-y)^2 = (10)(x^3)(y^2)$

4th $_5C_3\ (x)^2(-y)^3 = (10)(x^2)(-y^3)$

5th $_5C_4\ (x)^1(-y)^4 = (5)(x)(y^4)$

6th $_5C_5\ (x)^0(-y)^5 = (1)(1)(-y^5)$

$x^5-5x^4y+10x^3y^2-10x^2y^3+5xy^4-y^5$

4) Given $(2x+5)^{12}$, find the 4th term

$n = 12$ $\quad$ $r = \text{term} - 1$
$= 4 - 1$
$= 3$

4th term $\quad {}_{12}C_3 \, (2x)^{9} (5)^{3}$

$(220)(512x^9)(125)$

$14080000x^9$

5) Given $(x-3y)^{10}$, find the 8th term.

$n = 10$ $\quad$ $r = 8 - 1 = 7$

8th term $\quad {}_{10}C_7 \, (x)^3 (-3y)^7$

$(120)(x^3)(-2187y^7)$

$-262440x^3y^7$