

Arithmetic Sequences

A sequence that follows a pattern of adding or subtracting the same amount.

Common Difference (d) - The difference between successive terms in an arithmetic sequence.

Ex: $3, 8, 13, 18, \dots$ $d=5$
 $1, \frac{1}{2}, 0, -\frac{1}{2}, \dots$ $d=-\frac{1}{2}$

n th term of an Arithmetic Sequence (Formula)

$$a_n = a_1 + (n-1)d$$

a_n - n th term

a_1 = 1st term

n - # of terms

d - common difference

Ex: Find the formula for the n th term of the arithmetic sequence.

1) $d=-2$ $a_1=2$

$$a_n = a_1 + (n-1)d$$

$$a_n = 2 + (n-1)(-2)$$

$$a_n = 2 - 2n + 2 \quad a_n = 4 - 2n$$

2) $d=5$ $a_1=\frac{1}{2}$

$$a_n = \frac{1}{2} + (n-1)(5)$$

$$a_n = \frac{1}{2} + 5n - 5$$

$$a_n = 5n - 4.5$$

Ex: Answer the question about the arithmetic sequence.

$$1) d = -\frac{1}{2} \quad a_1 = -5 \quad a_{10} = \underline{-9.5}$$

$$a_n = a + (n-1)d$$

$$a_n = -5 + (n-1)\left(-\frac{1}{2}\right) \leftarrow n\text{th term}$$

$$a_{10} = -5 + (10-1)\left(-\frac{1}{2}\right)$$

$$a_{10} = -9.5$$

$$a_{10} = -5 + (10-1)\left(-\frac{1}{2}\right)$$

$$a_{10} = -5 + 9\left(-\frac{1}{2}\right)$$

$$a_{10} = -5 + (-4.5)$$

$$a_{10} = -9.5$$

$$2) d = 2 \quad a_6 = 15$$

$$a_9 = \underline{21}$$

$$a_n = a + (n-1)d$$

$$15 = a + (6-1)(2)$$

$$15 = a + (5)(2)$$

$$15 = a + 10$$

$$-10 \quad -10$$

$$\underline{a = 5}$$

$$n\text{th term: } a_n = 5 + (n-1)(2)$$

$$a_9 = 5 + (9-1)(2)$$

$$a_9 = 5 + (8)(2)$$

$$a_9 = 5 + 16$$

$$a_9 = 21$$

3) $d=4$ $a_8=35$ $a=7$

$n \rightarrow$ $\leftarrow a_n$

$$a_n = a + (n-1)d$$

$$35 = a + (8-1)(4)$$

$$35 = a + 7(4)$$

$$35 = a + 28$$

$$7 = a$$

$$a_n = 7 + (n-1)4$$

$$a_{100} = 403$$

$$a_{100} = 7 + (100-1)(4)$$

$$a_{100} = 403$$

4) Find the number of terms in the arithmetic sequence.

$+3$

\curvearrowright

$7, 10, 13, \dots, 55$

$a_1 \quad a_2 \quad a_3 \quad \dots \quad a_n$

$$a_n = a + (n-1)d$$

$$55 = 7 + (n-1)3$$

$$a_n = 55$$

$$55 = 7 + 3n - 3$$

$$a = 7$$

$$55 = 4 + 3n$$

$$n =$$

$$51 = 3n$$

$$d = 3$$

$$17 = n$$

5) The last term of an arithmetic sequence is 207. The common difference is 3, and the # of terms is 14. What is the first term.

$$\begin{array}{l}
 a_n = a + (n-1)d \\
 a_n = 207 \\
 a = \underline{\hspace{2cm}} \\
 n = 14 \\
 d = 3
 \end{array}
 \qquad
 \begin{array}{l}
 207 = a + (14-1)(3) \\
 207 = a + (13)(3) \\
 207 = a + 39 \\
 168 = a
 \end{array}$$

6) Given $a_3 = 14$ and $a_9 = -1$, find the first term, the common difference, and the n th term of the arithmetic sequence.

$$\begin{array}{l}
 \begin{array}{c} \leftarrow a_n \\ n \rightarrow \end{array} a_3 = 14 \\
 \begin{array}{c} \leftarrow a_n \\ n \rightarrow \end{array} a_9 = -1
 \end{array}
 \qquad
 \begin{array}{l}
 14 = a + (3-1)d \\
 14 = a + 2d
 \end{array}$$

$$\begin{array}{l}
 \begin{array}{c} \leftarrow a_n \\ n \rightarrow \end{array} a_9 = -1 \\
 \begin{array}{c} \leftarrow a_n \\ n \rightarrow \end{array} a_3 = 14
 \end{array}
 \qquad
 \begin{array}{l}
 -1 = a + (9-1)d \\
 -1 = a + 8d
 \end{array}$$

$$a = 19$$

$$d = \frac{-5}{2}$$

$$a_n = 19 + (n-1)\left(\frac{-5}{2}\right)$$

$$\begin{array}{r}
 a + 2d = 14 \\
 - (a + 8d = -1) \\
 \hline
 -6d = 15 \\
 \frac{-6d}{-6} = \frac{15}{-6} \\
 d = \frac{-5}{2}
 \end{array}$$

$$\begin{array}{r}
 a + 2d = 14 \\
 a + 2\left(\frac{-5}{2}\right) = 14 \\
 a + (-5) = 14 \\
 a - 5 = 14 \\
 a = 19
 \end{array}$$

7) Find the first term, common difference and nth term if $a_4 = -4$ and $a_7 = 23$

$$\begin{aligned}
 -4 &= a + (4-1)d & 23 &= a + (7-1)d & a &= -31 \\
 -4 &= a + 3d & 23 &= a + 6d & d &= 9 \\
 & & & & a_n &= -31 + (n-1)(9)
 \end{aligned}$$

$$\begin{array}{r}
 a + 3d = -4 \\
 - (a + 6d = 23) \\
 \hline
 -3d = -27 \\
 d = 9
 \end{array}
 \qquad
 \begin{array}{r}
 a + 3d = -4 \\
 a + 3(9) = -4 \\
 a + 27 = -4 \\
 a = -31
 \end{array}$$