13.1 Arithmetic and Geometric Sequences

Sequence: A set of numbers, called terms, arranged in some particular order.

Arithmetic Sequence: A sequence where the difference between each term is constant.

- Common difference - the difference between each term in an arithmetic sequence.

Ex: $2, 5, 8, 11, 14$ — Common difference: 3

$+3 \quad +3 \quad +3$

$12, 10, 8, 6, 4$ — Common difference: -2

$-2 \quad -2 \quad -2 \quad -2$

Geometric Sequence: A sequence where the ratio between each ~~term~~ term is constant.

- Common Ratio: the ratio between each term in a geometric sequence.

Ex: $2, 6, 18, 54$ — Common ratio - 3

$\times 3$

$20, 10, 5, \frac{5}{2}$ — Common ratio - $\frac{1}{2}$

$\times \frac{1}{2}$

Formulas:

Arithmetic – $t_n = t_1 + (n-1)d$

t_1 - 1st term d - common difference

t_2 - 2nd term

t_3 - 3rd term

⋮

t_n - nth term

Geometric – $t_n = t_1 r^{n-1}$

t_1 - 1st term r - common ratio

t_n - nth term

Examples) Find the first 4 terms and state if the sequence is arithmetic, geometric, or neither.

1) $t_n = 3n+2$

$t_1 = 3(1)+2 = 5$
$t_2 = 3(2)+2 = 8$
$t_3 = 3(3)+2 = 11$
$t_4 = 3(4)+2 = 14$

5, 8, 11, 14 arithmetic

2) $t_n = n^2+1$

$t_1 = (1)^2+1 = 2$
$t_2 = (2)^2+1 = 5$
$t_3 = (3)^2+1 = 10$
$t_4 = (4)^2+1 = 17$

2, 5, 10, 17 neither

3) $t_n = 3 \cdot 2^n$ 6, 12, 24, 48 geometric

Examples) Find a formula for each sequence.

1) 2, 5, 8, 11, 14 . . . (arithmetic)

$t_n = t_1 + (n-1)d$ $t_1 = 2$ $d = 3$
$t_n = 2 + (n-1)3$
$t_n = 3n-1$

2) 4, 8, 16, 32 . . . (geometric) $t_n = 2 \cdot 2 \cdot 2^{n-1}$ $t_n = 2 \cdot 2^n$

$t_n = t_1 r^{n-1}$ $t_1 = 4$ $r = 2$
$t_n = 4(2)^{n-1} \rightarrow t_n = 2 \cdot 2^n$

3) 21, 201, 2001, 20001, ... (neither)

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

20+1   200+1   2000+1   20000+1

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$2(10)+1 \quad 2(10)^2+1 \quad 2(10)^3+1 \quad 2(10)^4+1$

$$t_n = 2(10)^n + 1$$

Example: Find the indicated term of the arithmetic sequence. If $t_3 = 13$ and $t_7 = 29$, find t_{53}.

$$t_n = t_1 + (n-1)d$$

$t_3 = 13$ $t_7 = 29$

$t_n = 13$ $t_n = 29$

$n = 3$ $n = 7$

$13 = t_1 + (3-1)d$ $29 = t_1 + (7-1)d$

$13 = t_1 + 2d$ $29 = t_1 + 6d$

$\cancel{t} + 6d = 29$

$-(\cancel{t} + 2d = 13)$

$4d = 16$

$\boxed{d = 4}$

$t + 2(4) = 13$

$\boxed{t = 5}$

nth term

$$\boxed{\begin{aligned} t_n &= 5 + (n-1)4 \\ t_{53} &= 213 \end{aligned}}$$