

Sectors of a Circle

Sector - The region bounded by 2 radii and the intercepted arc.

r - radius

θ - central angle

s - arc length

K - area of the sector



Arc Length

$$C = 2\pi r$$

Area of a sector

$$A = \pi r^2$$

(Degrees) $S = \left(\frac{\theta}{360}\right) 2\pi r$

$K = \left(\frac{\theta}{360}\right) \pi r^2$ (Degrees)

$S = \left(\frac{\theta}{2\pi}\right) 2\pi r$

$K = \left(\frac{\theta}{2\pi}\right) \pi r^2$

(Radians) $S = \theta r$

$K = \frac{1}{2} \theta r^2$ (Radians)

$K = \frac{1}{2} \theta \cdot r$

$K = \frac{1}{2} r s$ (radiuses)

24 = 18θ
24 = θ
18
4 = θ
3

- 1) Find the arc length and area of a sector of a circle with radius 6 cm and central angle 55° .

$$S = \left(\frac{55}{360}\right) 2\pi(6) \quad K = \left(\frac{55}{360}\right) \pi (6)^2$$

$$= \frac{11\pi}{6} \text{ cm}$$

$$= 5.76 \text{ cm}$$

$$= \frac{55\pi}{10}$$

$$= \frac{11\pi}{2} \text{ cm}^2$$

$$= 17.28 \text{ cm}^2$$

- 2) A sector of a circle has an area of 24 square cm and a radius of 6 cm. Find the measure of the central angle in radians and degrees.

$$K = \frac{1}{2} r^2 \theta$$

$$\frac{4 \text{ rad}}{3} = \frac{180 \text{ deg}}{\pi \text{ rad}} = \boxed{76.4^\circ}$$

$$24 = \left(\frac{1}{2}\right) (6)^2 \theta$$

$$24 = 18 \theta$$

$$\frac{24}{18} = \theta$$

$$\boxed{\frac{4}{3} = \theta}$$

Linear and Angular Speed

- 1) A pulley rotates 210° every second. Through how many revolutions does it turn in one hour?

$$\frac{\text{degrees}}{\text{second}} \rightarrow \frac{\text{revolutions}}{\text{hour}}$$

$$1 \text{ revolution} = 360^\circ = 2\pi$$

$$\frac{210 \text{ deg}}{1 \text{ sec}} \left| \frac{1 \text{ rev}}{360 \text{ deg}} \right| \frac{60 \text{ sec}}{1 \text{ min}} \left| \frac{60 \text{ min}}{1 \text{ hr}} \right|$$

$$2100 \text{ rev/hr}$$

- 2) A bicycle wheel with a diameter of 24 in. travels at 33 rpm. Find the distance that a point on the wheel travels in 1 min.

$$\frac{33 \text{ rev}}{1 \text{ min}} \left| \frac{2\pi(12) \text{ in}}{1 \text{ rev}} \right| = 2488.14 \text{ in/min}$$

- 3) An electrical wind generator has propeller blades that are 5 m long. If the blades are rotating at 8π r/s, what is the linear velocity of a point on the tip of the blades?

$$\frac{8\pi \text{ rad}}{1 \text{ sec}} \left| \frac{1 \text{ rev}}{2\pi \text{ rad}} \right| \frac{2\pi(5) \text{ m}}{1 \text{ rev}} = 125.66 \text{ m/s}$$