

4.2 Operations With Functions

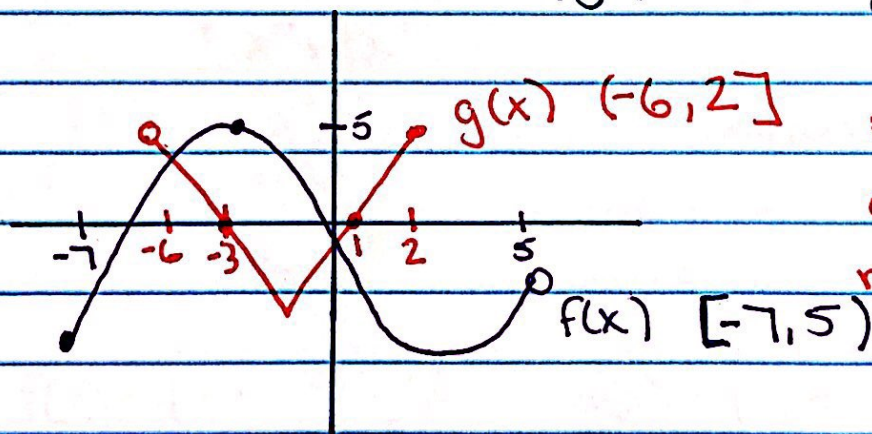
Given 2 functions $f(x)$ and $g(x)$, the following are true.

Sum of f and g $(f+g)(x) = f(x) + g(x)$

Difference of f and g $(f-g)(x) = f(x) - g(x)$

Product of f and g $(f \cdot g)(x) = f(x) \cdot g(x)$

Quotient of f and g $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$



Domain: The intersection excluding any values that make the denominator zero

$$\left(\frac{f}{g}\right)(-3) = \frac{f(-3)}{g(-3)} = \frac{0}{0} \text{ undefined}$$

Domain of $\left(\frac{f}{g}\right)(x) =$

Intersection: $(-6, 2]$

What makes $g(x) = 0$? $x = -3$ $x = 1$

$$(-6, -3) \cup (-3, 1) \cup (1, 2]$$

Domain is the intersection of the domains of f and g where both exist

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, $(\frac{f}{g})(x)$,
and their domains.

$$1) \quad f(x) = x^2 \quad g(x) = x+2$$

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$$(f+g)(x) = x^2 + (x+2) \quad \text{Domain: } (-\infty, +\infty)$$

$$(f-g)(x) = x^2 - (x+2) = x^2 - x - 2 \quad \text{Domain: } (-\infty, +\infty)$$

$$(f \cdot g)(x) = x^2(x+2) = x^3 + 2x^2 \quad \text{Domain: } (-\infty, +\infty)$$

$$\left(\frac{f}{g}\right)(x) = \frac{x^2}{x+2} \quad \text{Domain: } (-\infty, -2) \cup (-2, +\infty)$$

$x \neq -2$

$$2) \quad f(x) = \sqrt{1+x^2} \quad \mathbb{R} \quad g(x) = \sqrt{1-x} \quad \begin{matrix} 1-x \geq 0 \\ 1 \geq x \\ (-\infty, 1] \end{matrix}$$

$$(f+g)(x) = \sqrt{1+x^2} + \sqrt{1-x} \quad D: (-\infty, 1]$$

$$(f-g)(x) = \sqrt{1+x^2} - \sqrt{1-x} \quad D: (-\infty, 1]$$

$$(f \cdot g)(x) = \sqrt{(1+x^2)(1-x)} \quad D: (-\infty, 1]$$

$$\left(\frac{f}{g}\right)(x) = \sqrt{\frac{1+x^2}{1-x}} \quad D: (-\infty, 1)$$

Evaluating Functions

$$f(x) = x + 4$$

1) Find $f(3)$

$$f(3) = 3 + 4$$

$$f(3) = 7$$

2) Find $f(a+2)$

$$f(a+2) = (a+2) + 4$$

$$= a + 6$$

$$g(x) = x^2 + 1$$

1) $g(x+2) = (x+2)^2 + 1$
 $= x^2 + 4x + 4 + 1$
 $= x^2 + 4x + 5$

2) $g(x) + g(2) = x^2 + 1 + 2^2 + 1$
 $= x^2 + 6$

Completing the Square

1) $y = x^2 - 6x + 14$

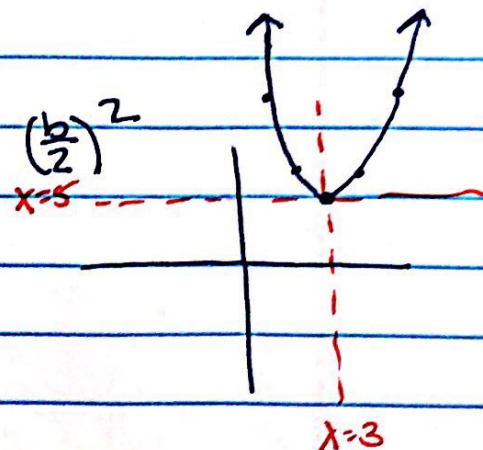
$$y - 14 = x^2 - 6x$$

$$y - 14 + 9 = x^2 - 6x + 9$$

$$y - 5 = (x - 3)^2$$

$$y = (x - 3)^2 + 5$$

Vertex: $(3, 5)$



$$2) y = 2x^2 + 4x - 1$$

$$y + 1 = 2x^2 + 4x$$

$$y + 1 = 2(x^2 + 2x)$$

$$y + 1 + \underline{2(1)} = 2(x^2 + 2x + \underline{1})$$

$$y + 3 = 2(x + 1)^2$$

$$y = 2(x + 1)^2 - 3$$